Uncloneable states are necessary as proofs and advice

Rohit Chatterjee, **Srijita Kundu** & Supartha Podder

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- Secure software leasing: Ananth and La Placa (2021), Broadbent, Jeffery, Lord, Podder and Sundaram (2021), ...

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3. There is a relational problem which can only be exactly solved by a quantum poly-time algorithm with an uncloneable quantum state as quantum advice.

Quantum proofs

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Quantum generalization of NP

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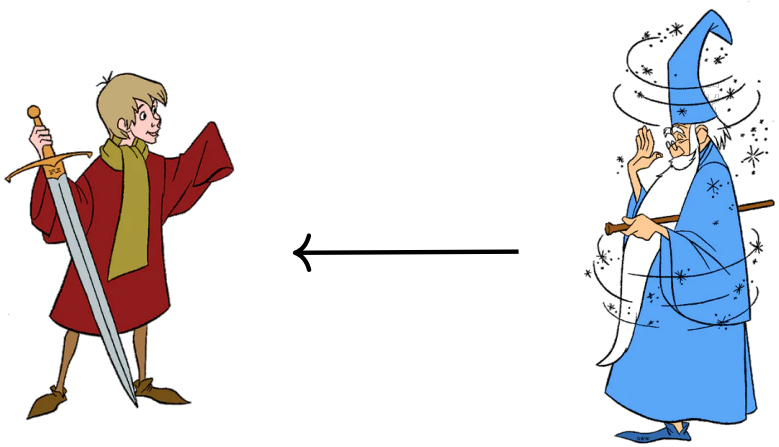

Input $x \in \{0,1\}^*$, is $x \in$ YES?

Knows x

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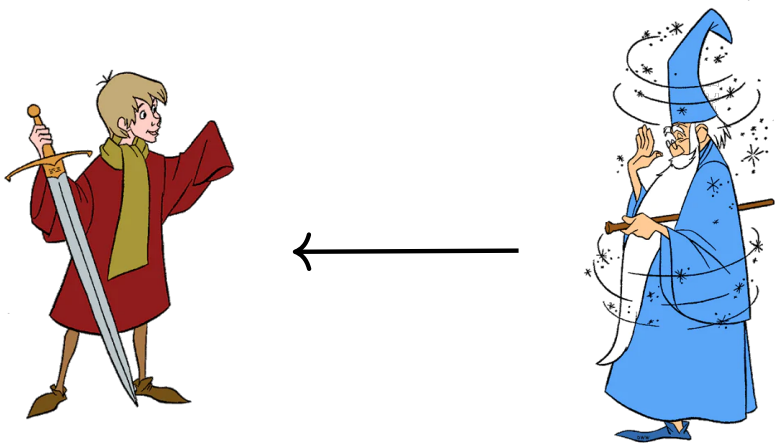
Quantum generalization of NP

- If YES instance, exists witness that can convince Arthur w.h.p.
- If NO instance, every witness is rejected by Arthur w.h.p.



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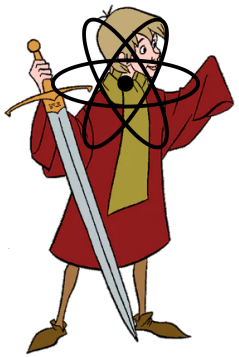
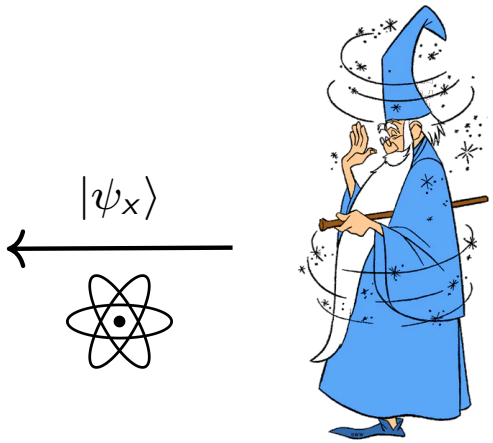


$|\psi_x\rangle$



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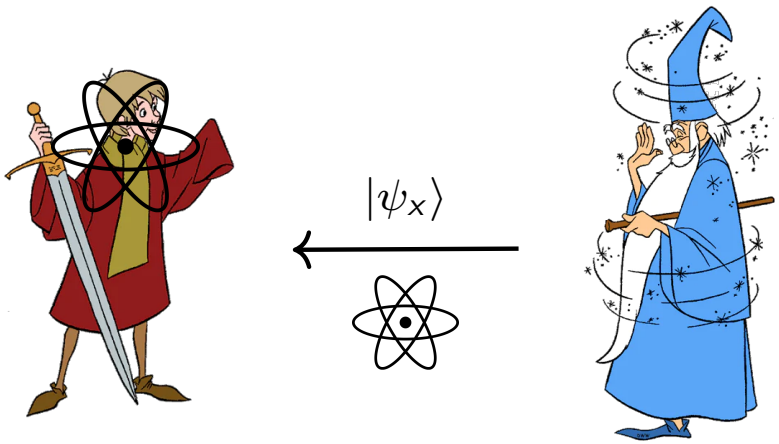

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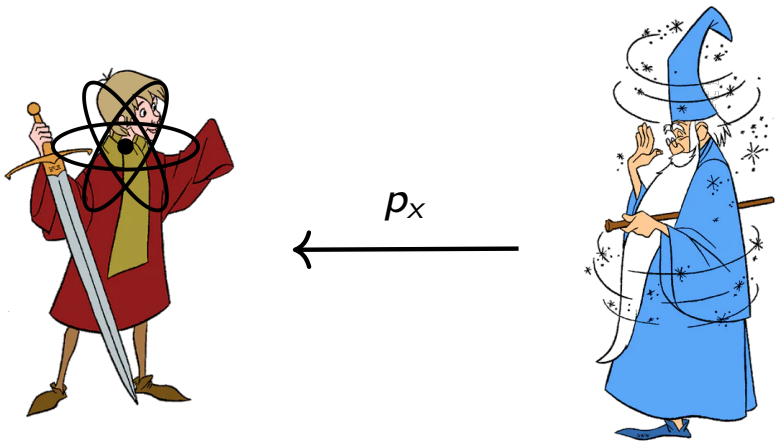

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- Classical proofs are always cloneable
- We use the AK07 quantum oracle

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- **FBQP/qpoly**: like BQP/qpoly for polynomially bounded relations $R \subseteq \{0,1\}^* \times \{0,1\}^*$, ...
- In fact the ABK23 construction separates FEQP/qpoly and FBQP/poly
- We use the ABK23 construction

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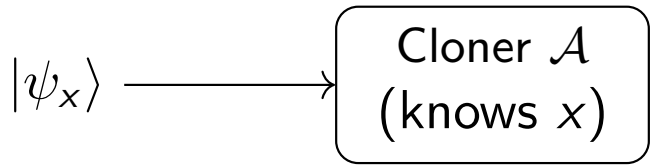

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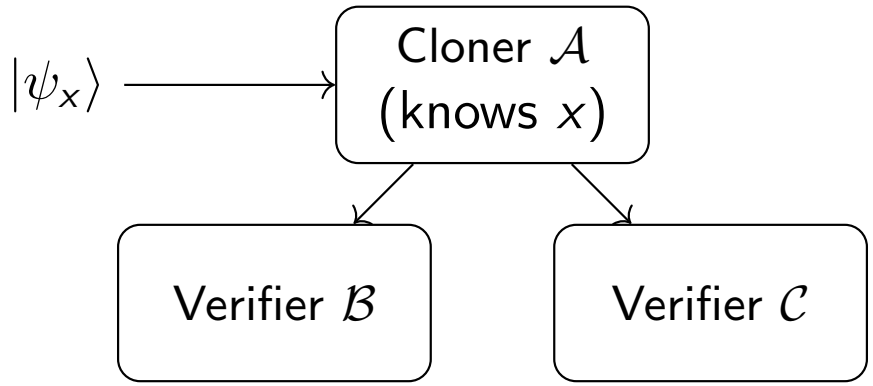

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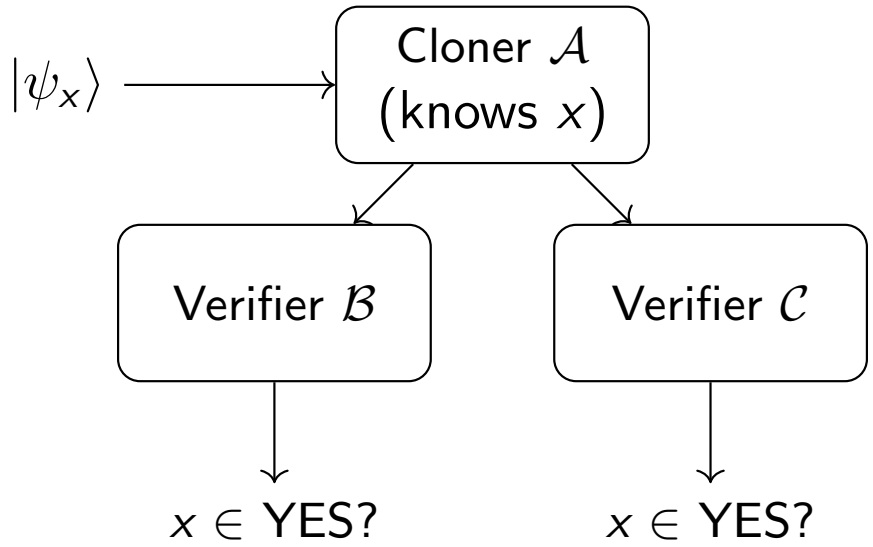

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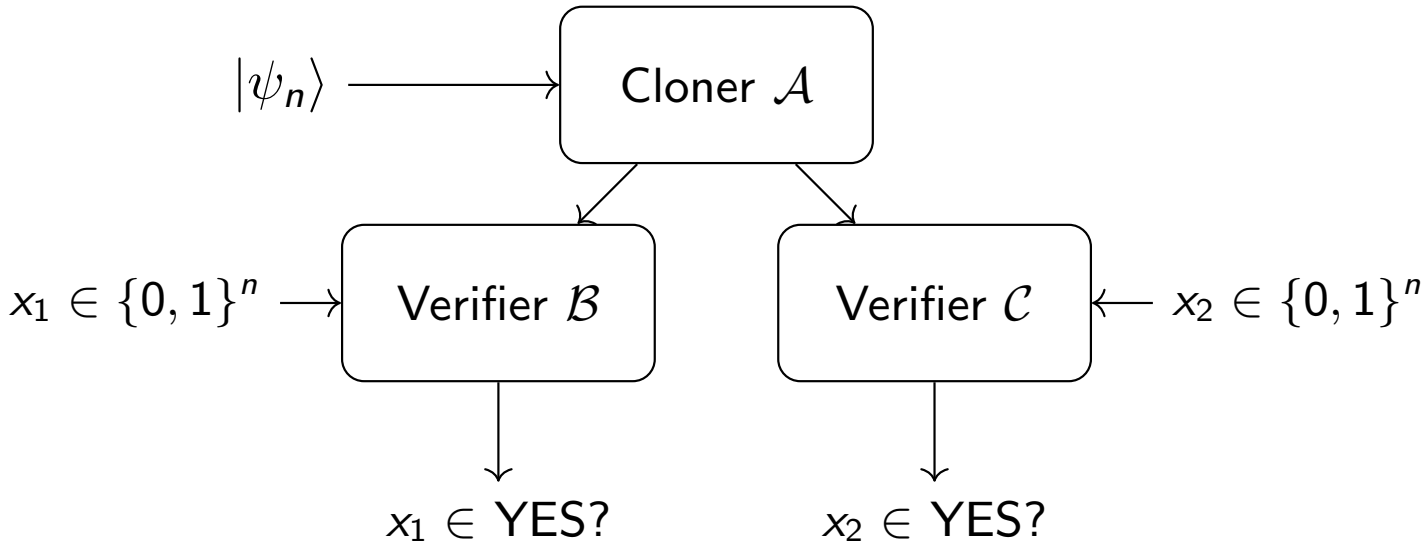

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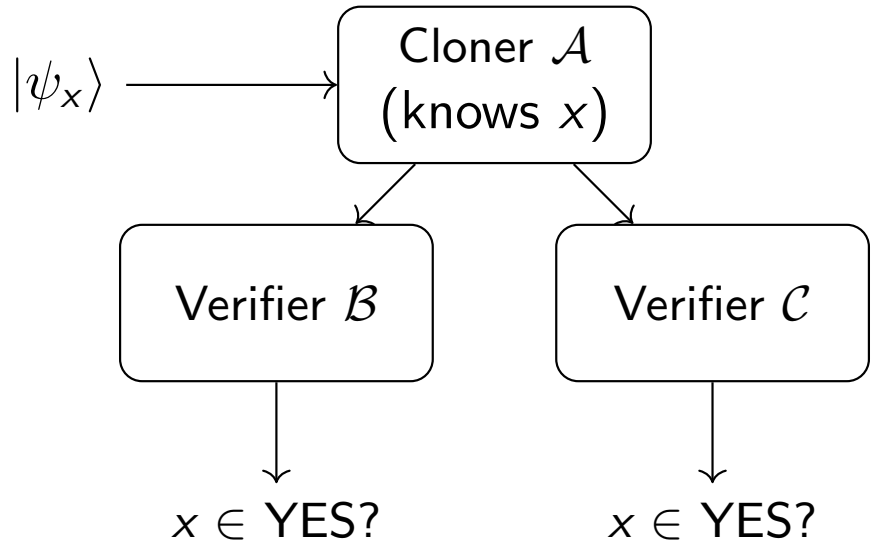

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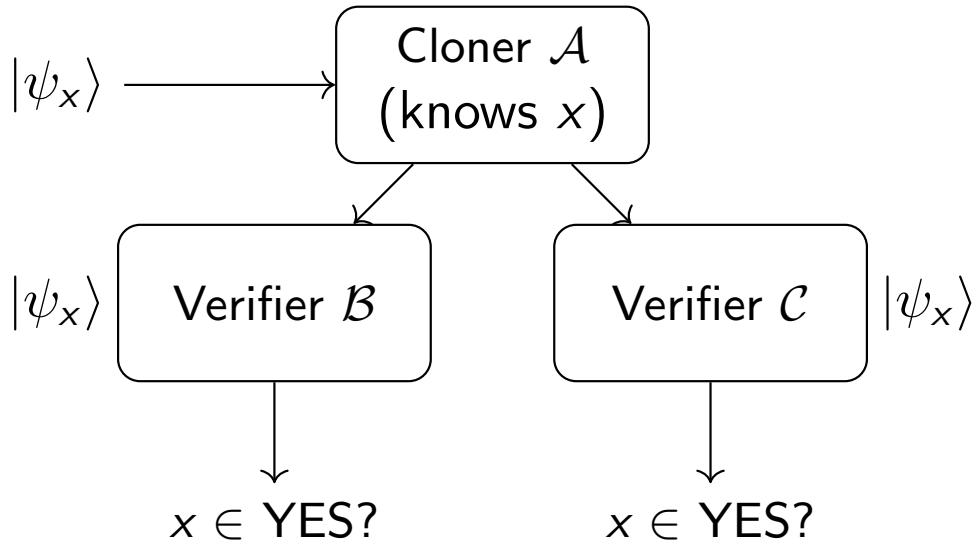

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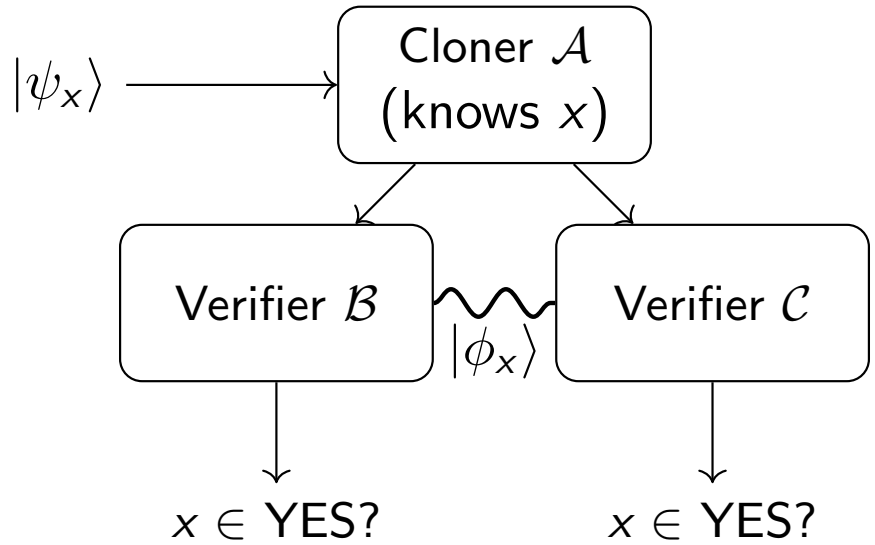

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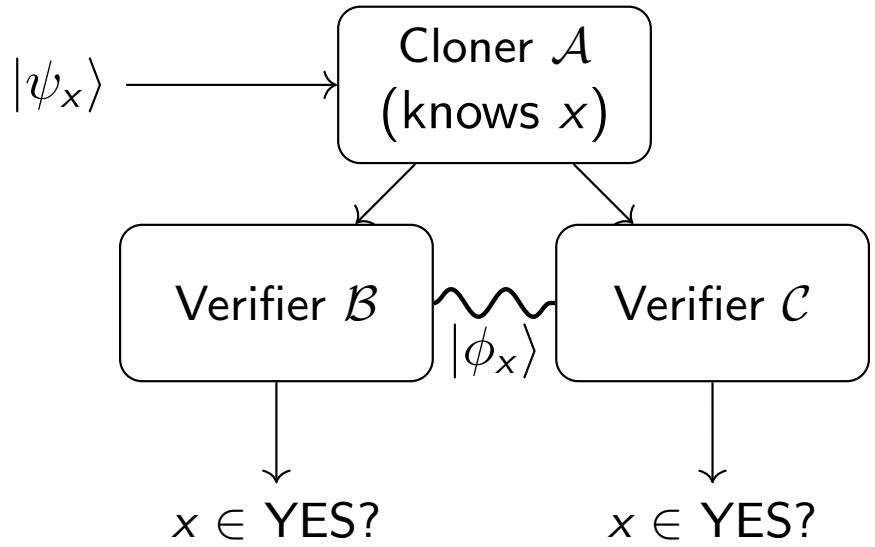

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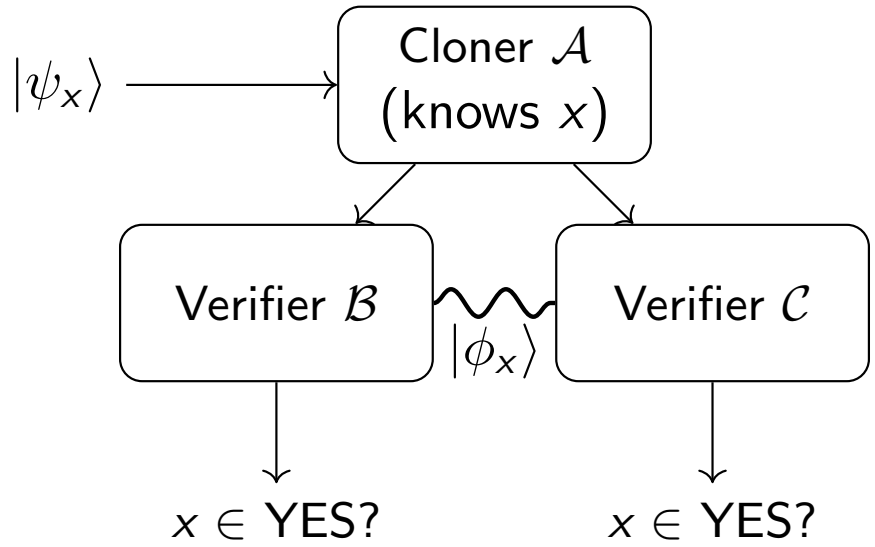

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- Often average-case uncloneability is considered

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- Uncloneability is worst-case instead of average case

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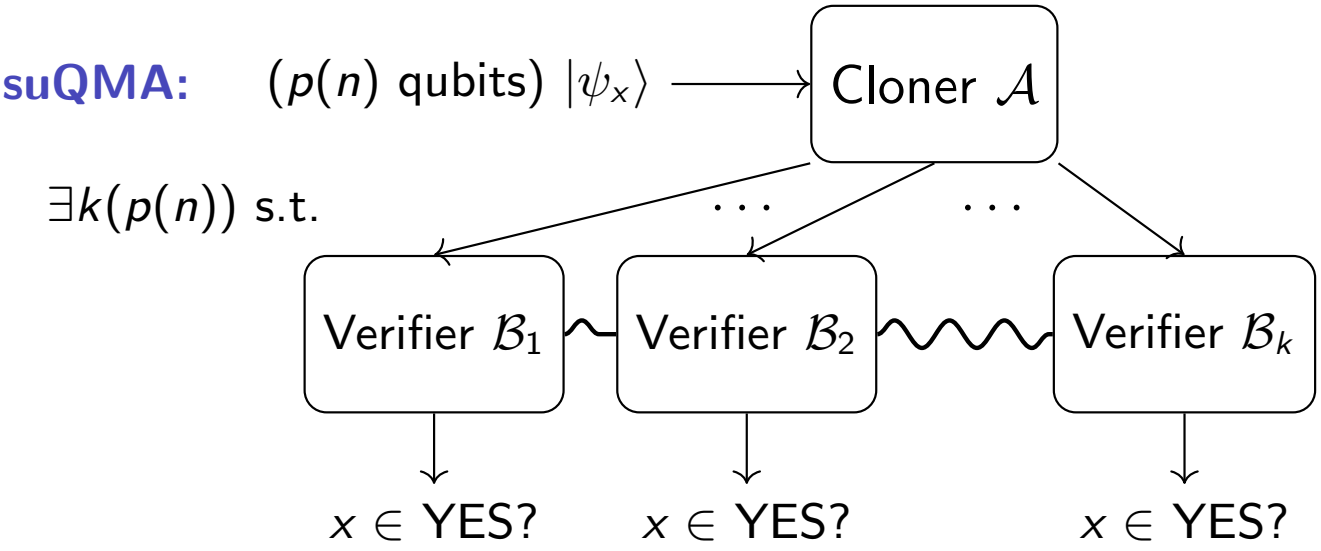

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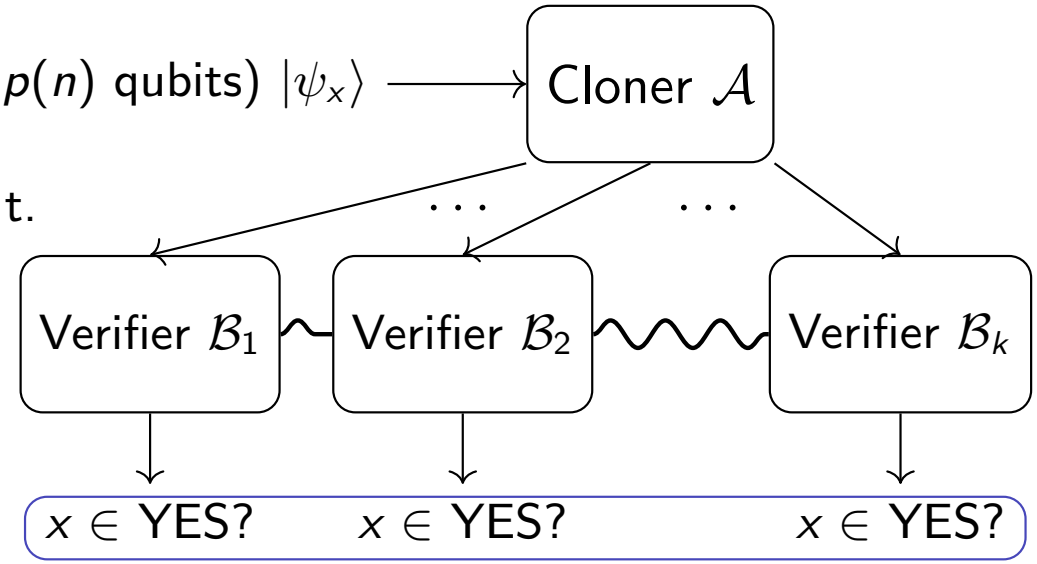

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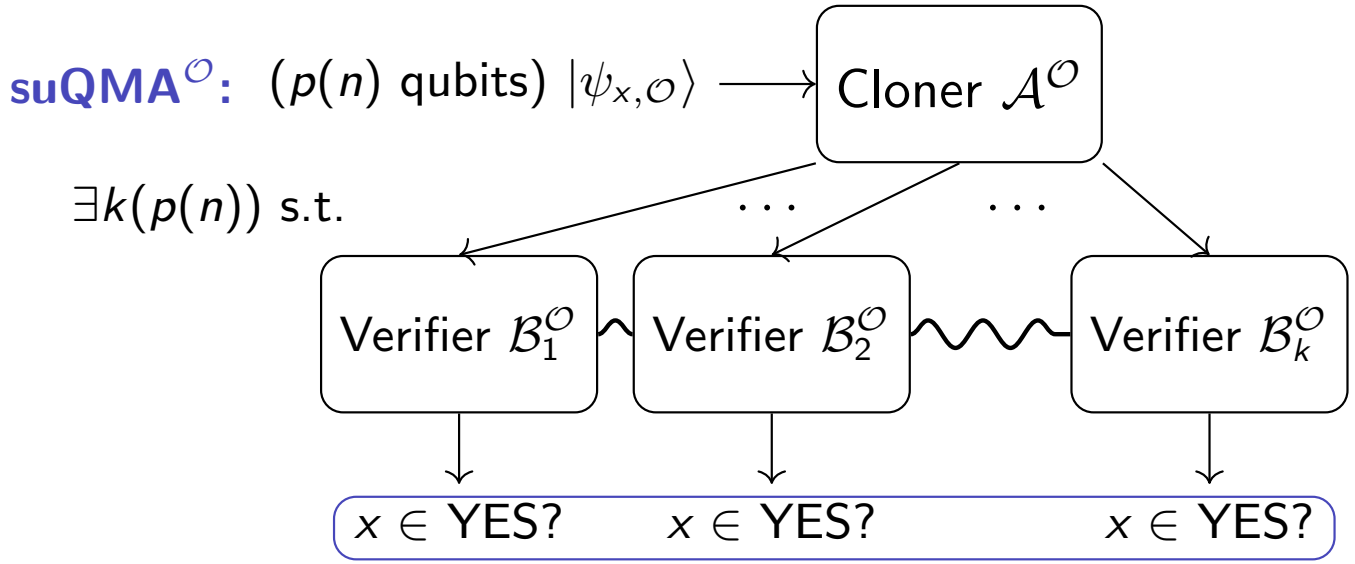

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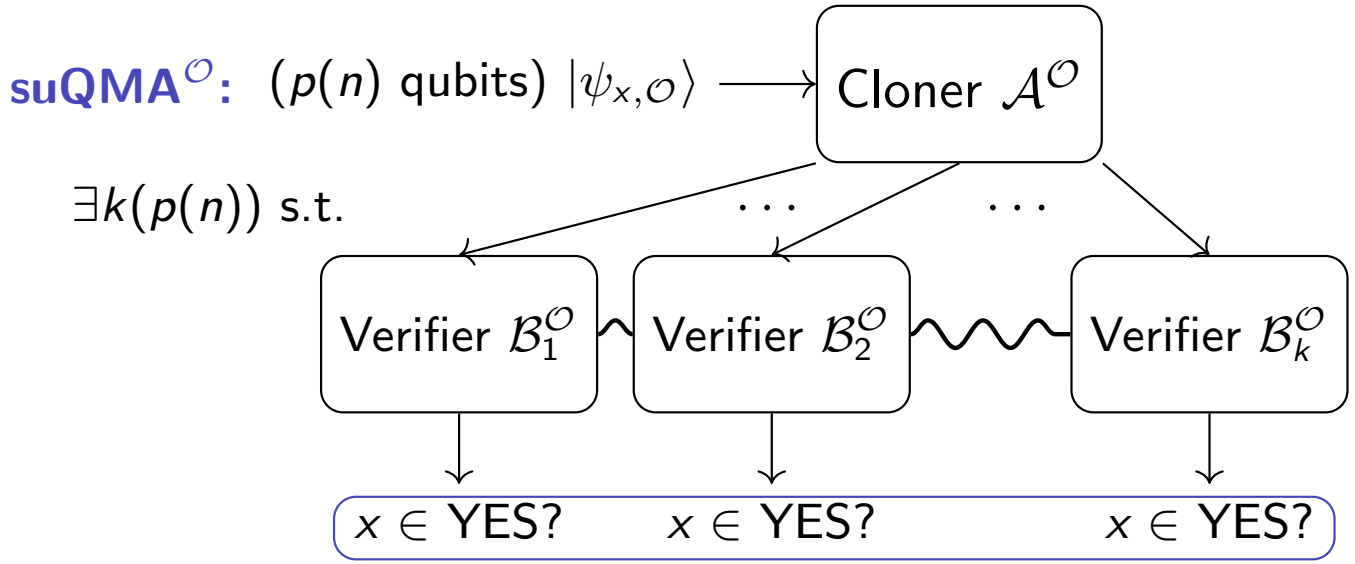

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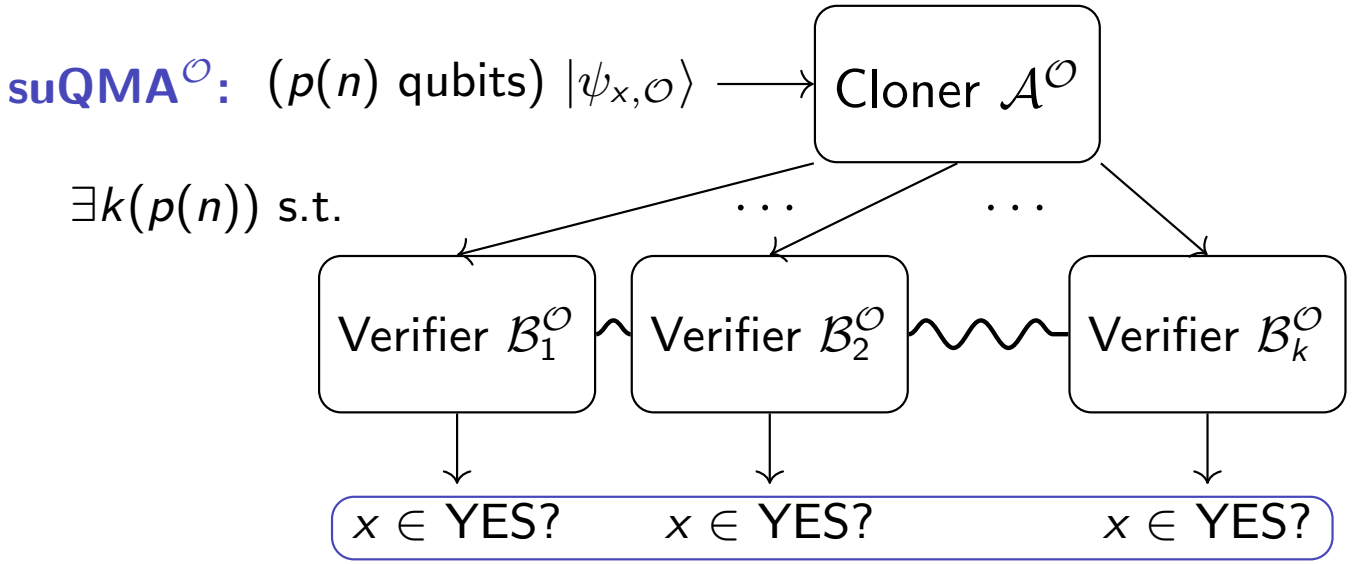

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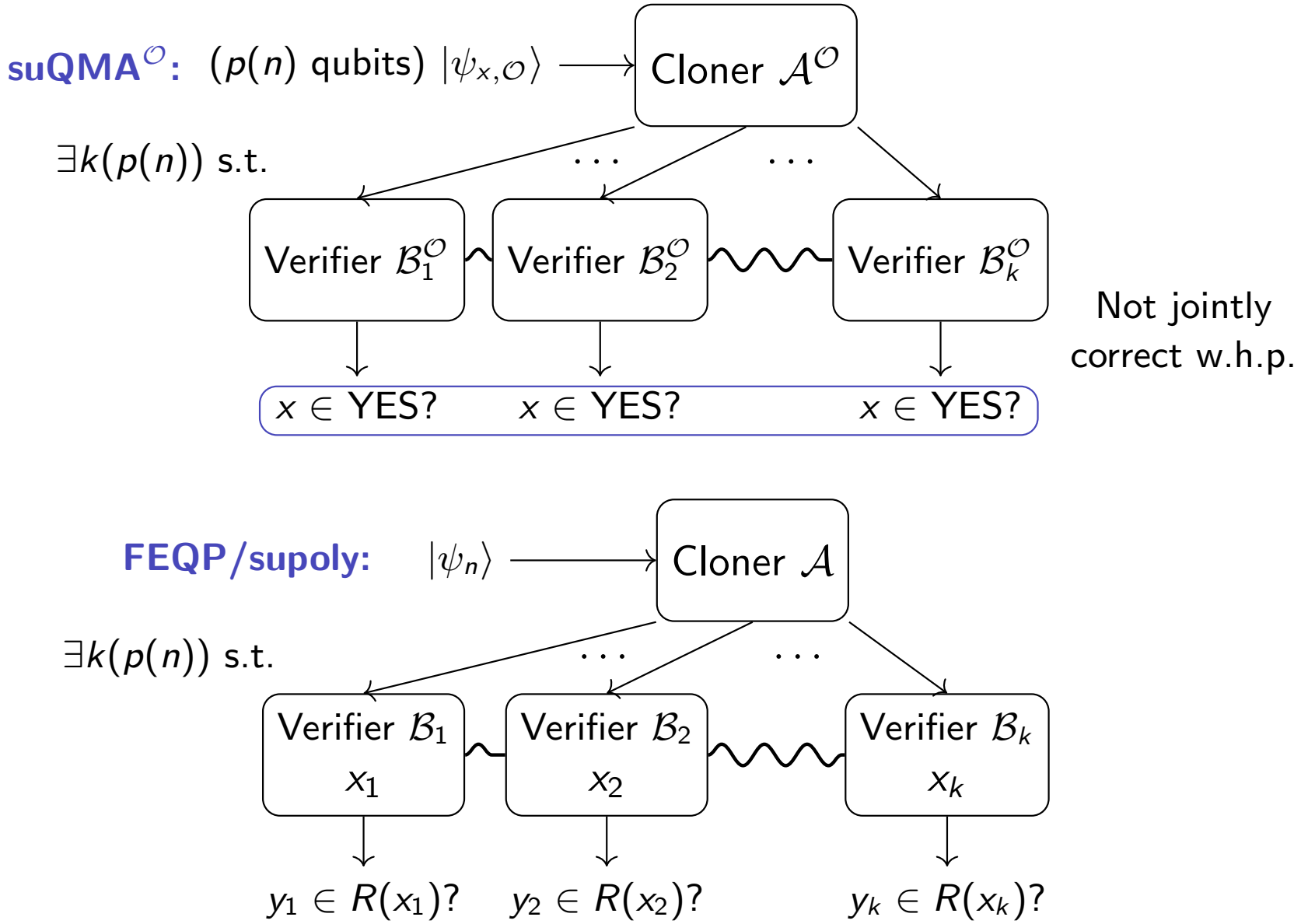

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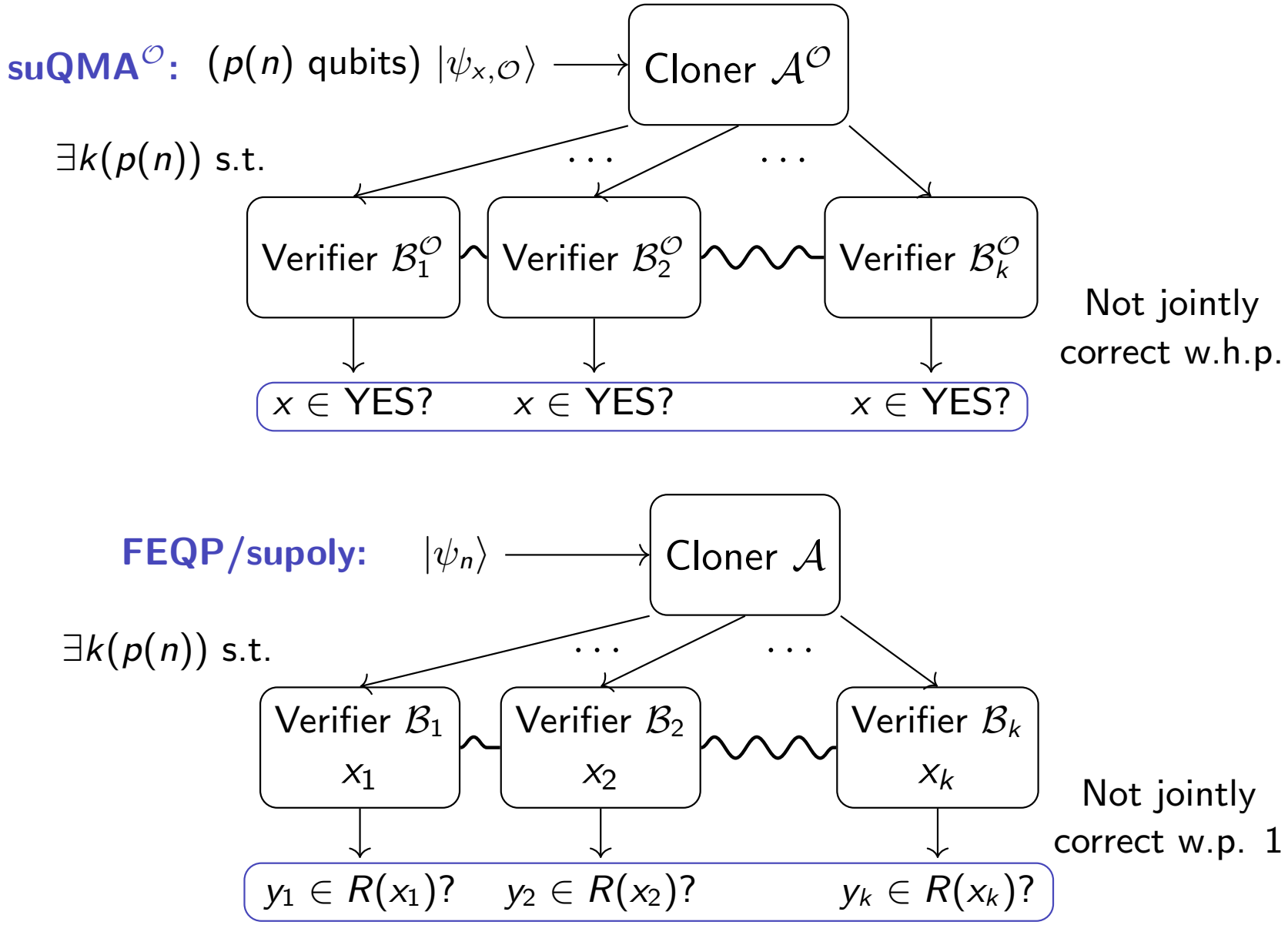

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AK07 oracle problem (QuantumOR): Given either $\mathbb{1} - 2|\psi\rangle\langle\psi|$ for Haar-random $|\psi\rangle$ (YES) or $\mathbb{1}$ (NO), determine which is the case

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FEQP/supoly:

ABK23 problem (Hidden Matching): Family of relations $\{R_f\}_f \subseteq \{0,1\}^n \times \{0,1\}^{n+1}$ indexed by $f : \{0,1\}^n \to \{0,1\}$

$$(x,(y,b)) \in R_f \text{ iff } y \in \{0,1\}^n \text{ and } f(y) \oplus f(y \oplus x) = b$$

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AK07 oracle problem (QuantumOR): Given either $\mathbb{1} - 2|\psi\rangle\langle\psi|$ for Haar-random $|\psi\rangle$ (YES) or $\mathbb{1}$ (NO), determine which is the case

Canonical proof: The state $|\psi\rangle$ for oracle $\mathbb{1} - 2|\psi\rangle\langle\psi|$

Good news: Haar-random states are uncloneable!

FEQP/supoly:

ABK23 problem (Hidden Matching): Family of relations $\{R_f\}_f \subseteq \{0,1\}^n \times \{0,1\}^{n+1}$ indexed by $f : \{0,1\}^n \to \{0,1\}$

$$(x,(y,b)) \in R_f \text{ iff } y \in \{0,1\}^n \text{ and } f(y) \oplus f(y \oplus x) = b$$

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Good news: Binary phase states (for random f) are uncloneable!

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   - For FEQP/supoly, we show:
     - Proofs $|\psi_f\rangle$ that works jointly for $\mathcal{B}_1, \ldots, \mathcal{B}_k$ must have inner product $|\langle\psi_{f'}|\psi_f\rangle| = 2^{-\Omega(k)}$;
     - In order to achieve the inner products, the states need $\Omega(k)$ qubits.

Other consequences

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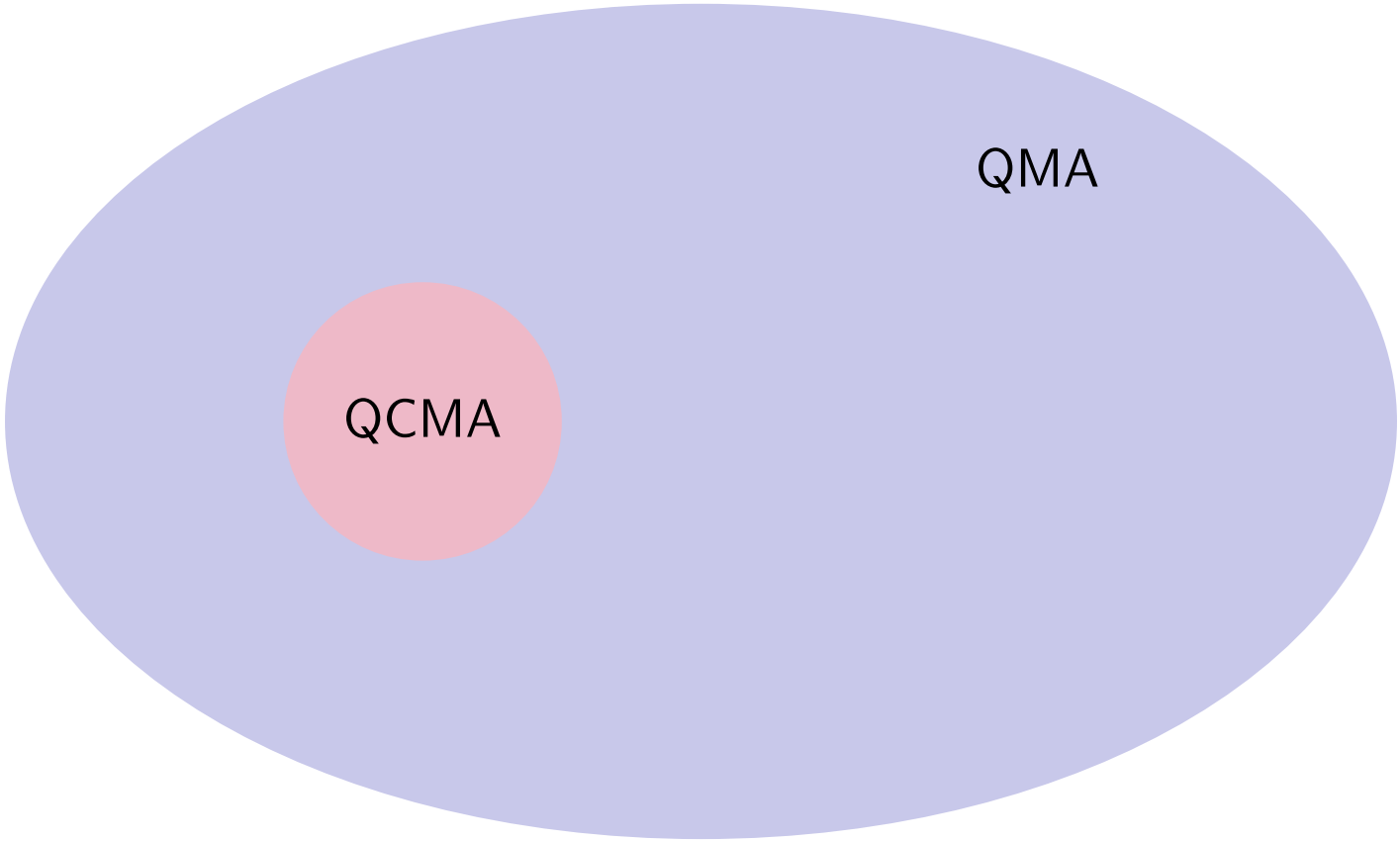

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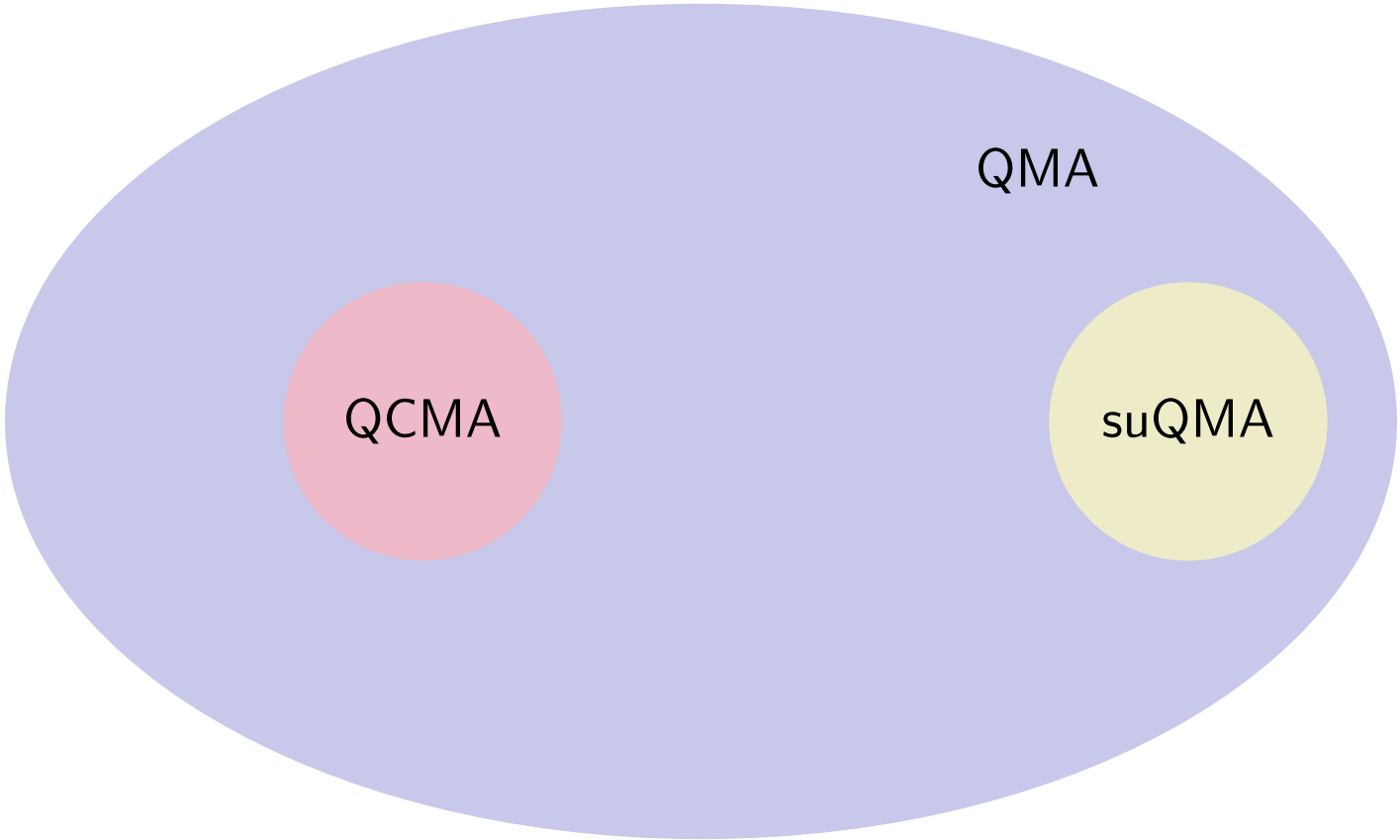

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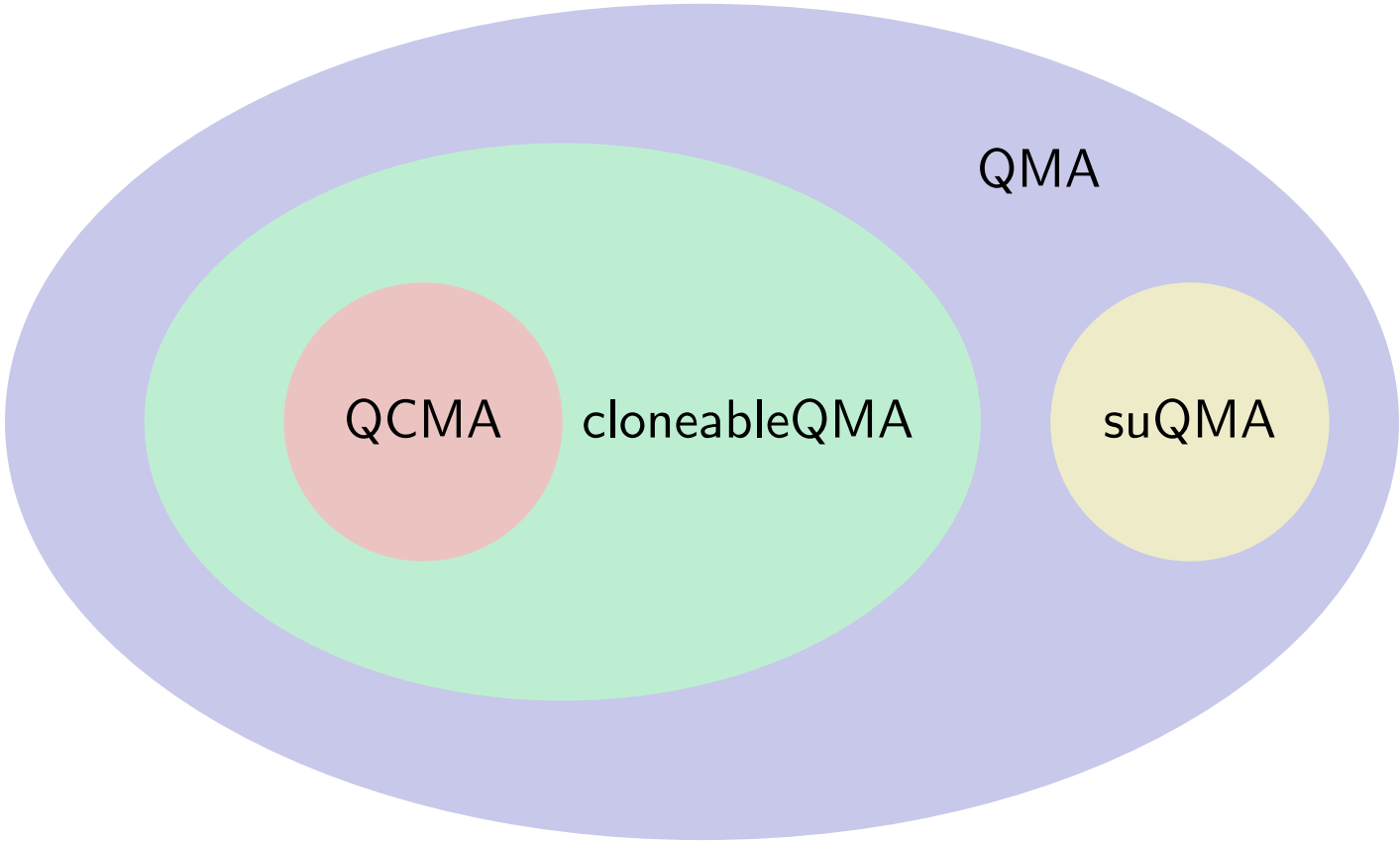

Nehoran and Zhandry (2023): w.r.t. a quantum oracle, cloneableQMA $\neq$ QCMA

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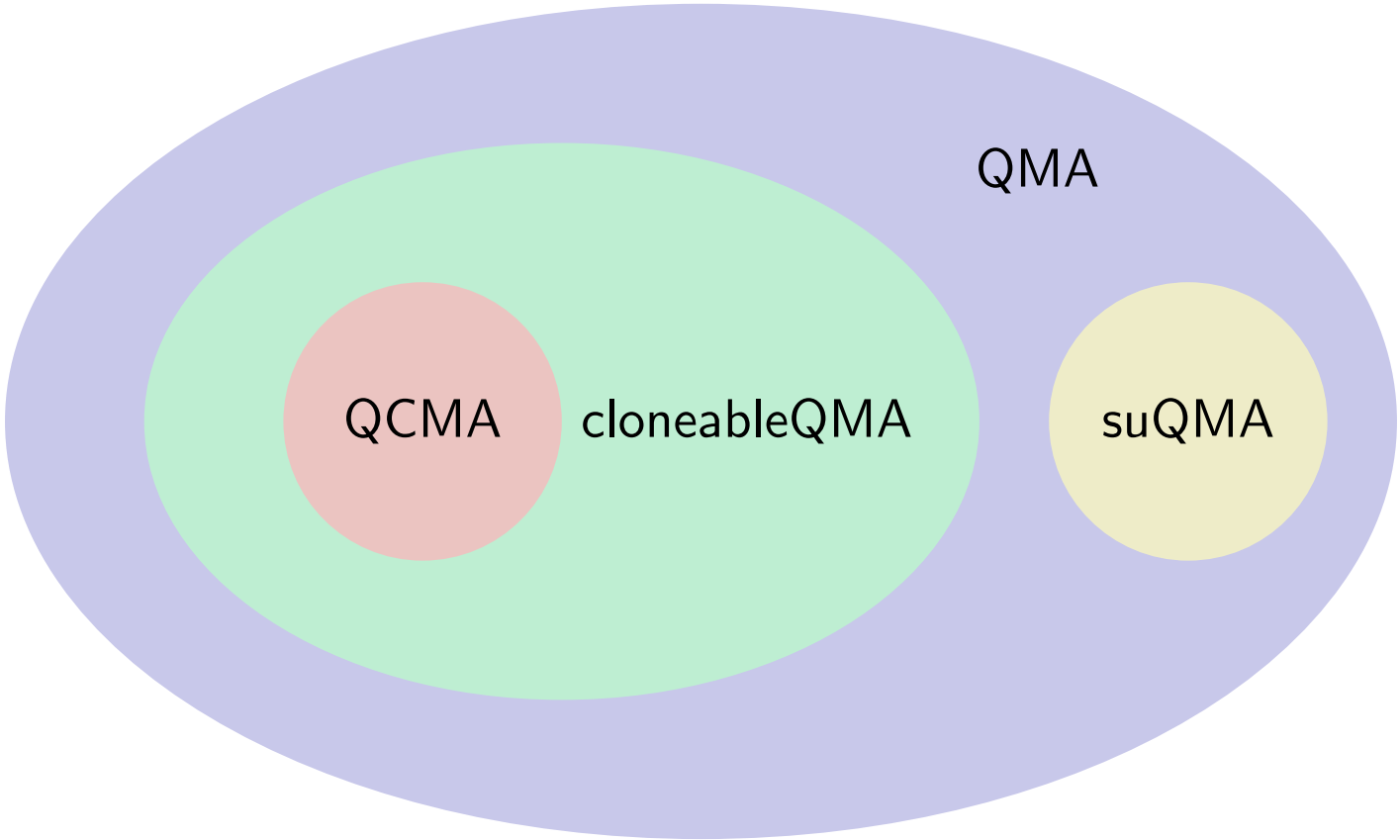

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From definition, suQMA $\subseteq$ QMA $\setminus$ cloneableQMA
$\Rightarrow$ w.r.t. a quantum oracle, QMA $\neq$ cloneableQMA

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