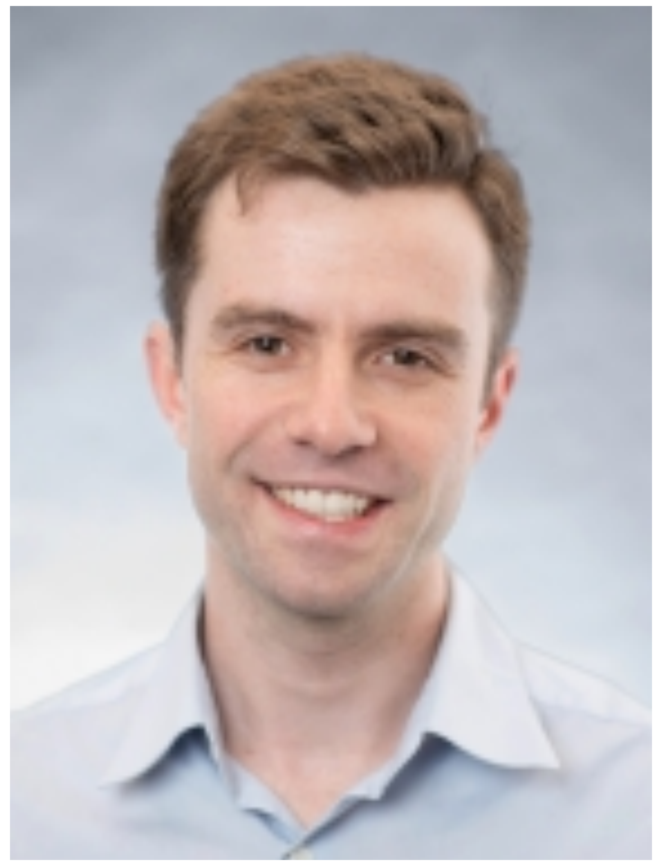
SNARGs for NP & Non-Signaling PCPs, Revisited

Surya Mathialagan

MIT → **NTT Research**

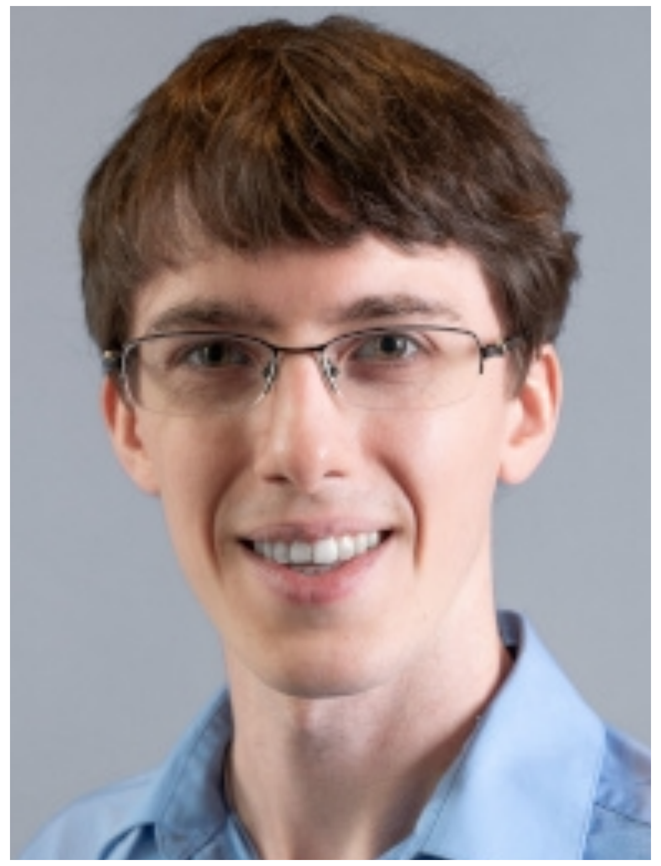
Lali Devadas
MIT



Sam Hopkins
MIT

Yael Kalai
MIT

Pravesh Kothari
Princeton



Alex Lombardi
Princeton


WANTED
DEAD OR ALIVE
CASH REWARD
$ 10.000


WANTED
PROVEN
OR ALIVE
CASH
REWARD
$ 10.000


WANTED
PROVEN
OR
DISPROVEN
CASH
REWARD
$ 10.000

WANTED

PROVEN OR **DISPROVEN**

CASH REWARD $ 10 **+ COOKIES**


WANTED
PROVEN
OR
DISPROVEN
Low-Norm
Nullstellensatz
Conjecture
CASH
REWARD
$ 10
+ COOKIES

TLDR

- **Theorem.** We construct SNARGs for NP assuming:
  - Hardness of LWE, Bilinear Maps or DDH,
  - A mathematical conjecture above multivariate polynomials of reals.
- **This talk:** I will talk about this fascinating connection between SNARGs and PCPs [BMW98, KRR14, BHK17, BKKSW18]
  - Giving you an open problem to solve :)

Delegation of Computation

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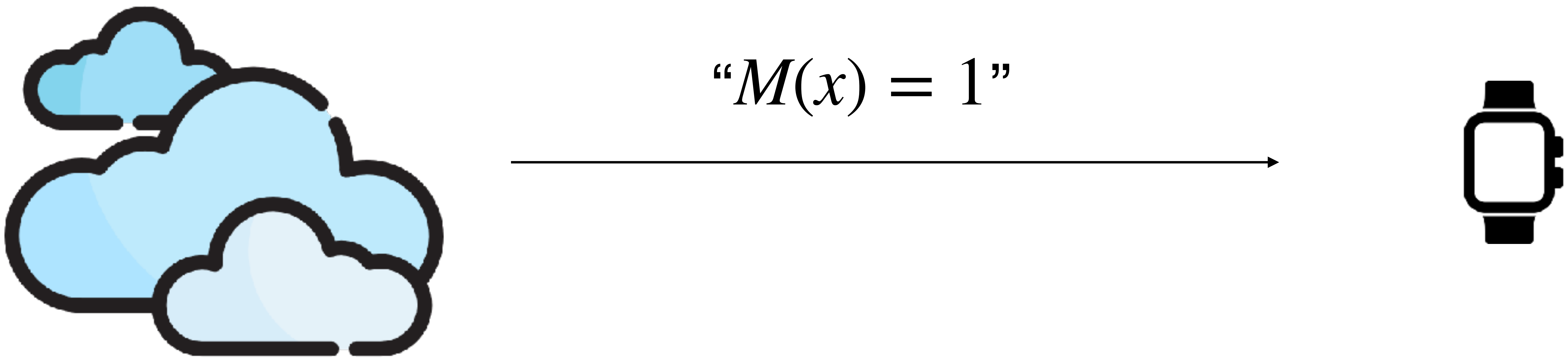
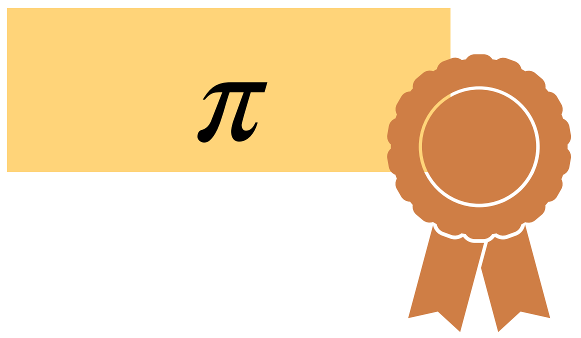


Delegation of Computation

“$M(x) = 1$”



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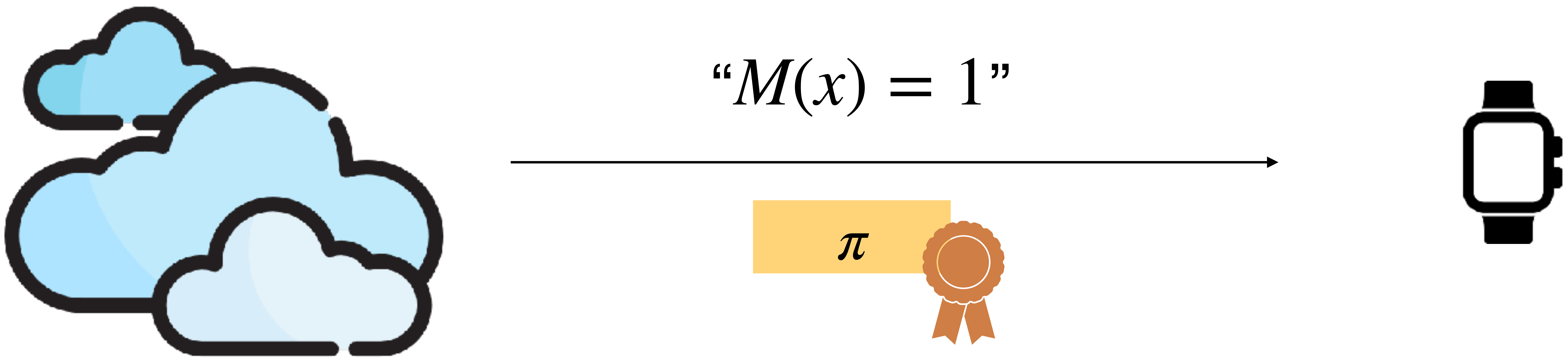

Can the cloud attach a **small, efficiently verifiable proof** that he did the computation correctly?

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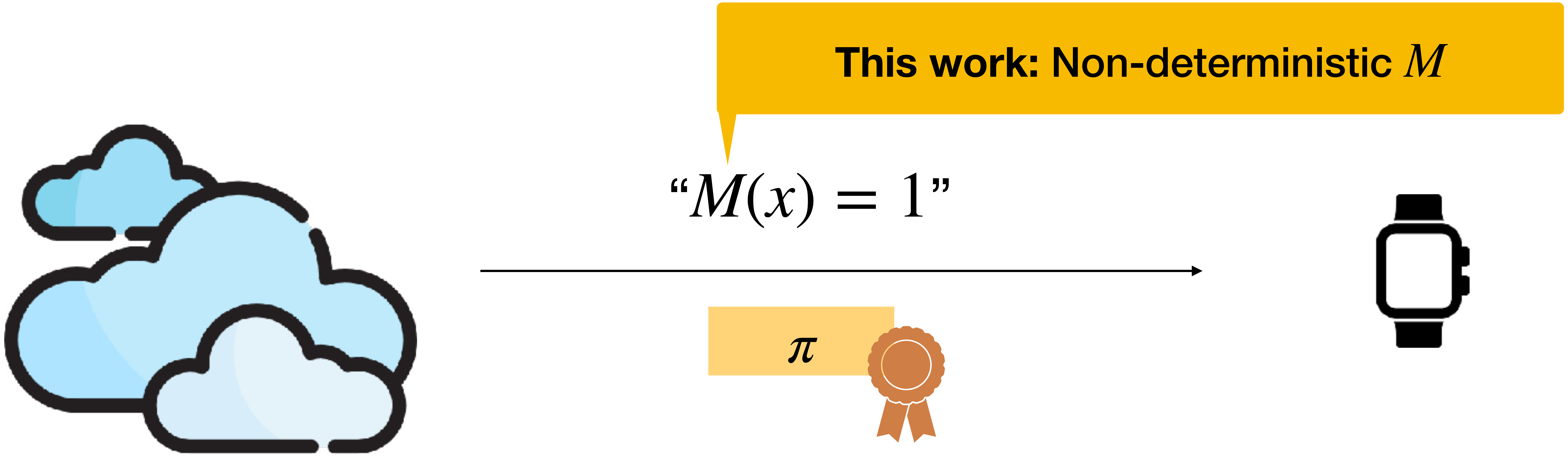

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Succinct **N**on-Interactive **Arg**uments for NP

Succinct Non-Interactive Arguments for NP

$\mathcal{P}$

Succinct Non-Interactive Arguments for NP

"$x \in \mathscr{L}$"

$\mathscr{P}$

Succinct Non-Interactive Arguments for NP

“$x \in \mathscr{L}$”

$\mathscr{P}$ 

 $\mathscr{V}$

Succinct Non-Interactive Arguments for NP

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$\mathscr{P}$ 

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w

Succinct Non-Interactive Arguments for NP

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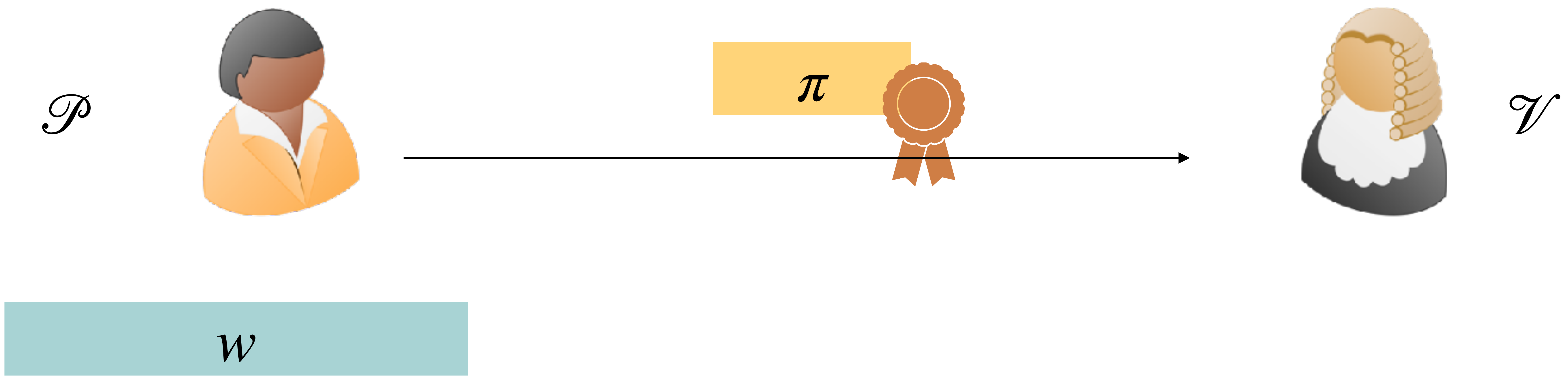

<u>S</u>uccinct <u>N</u>on-Interactive <u>Arg</u>uments for NP

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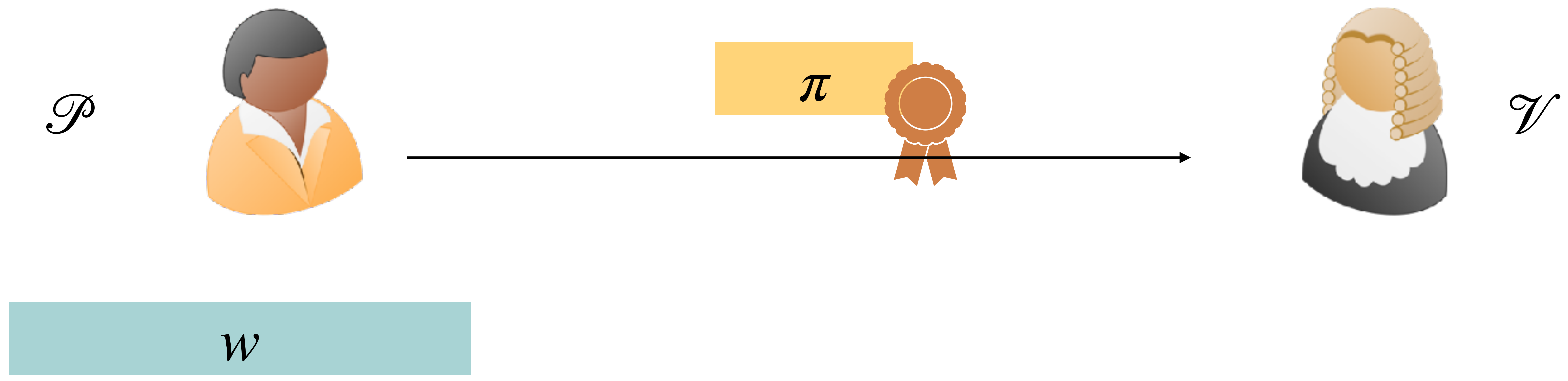
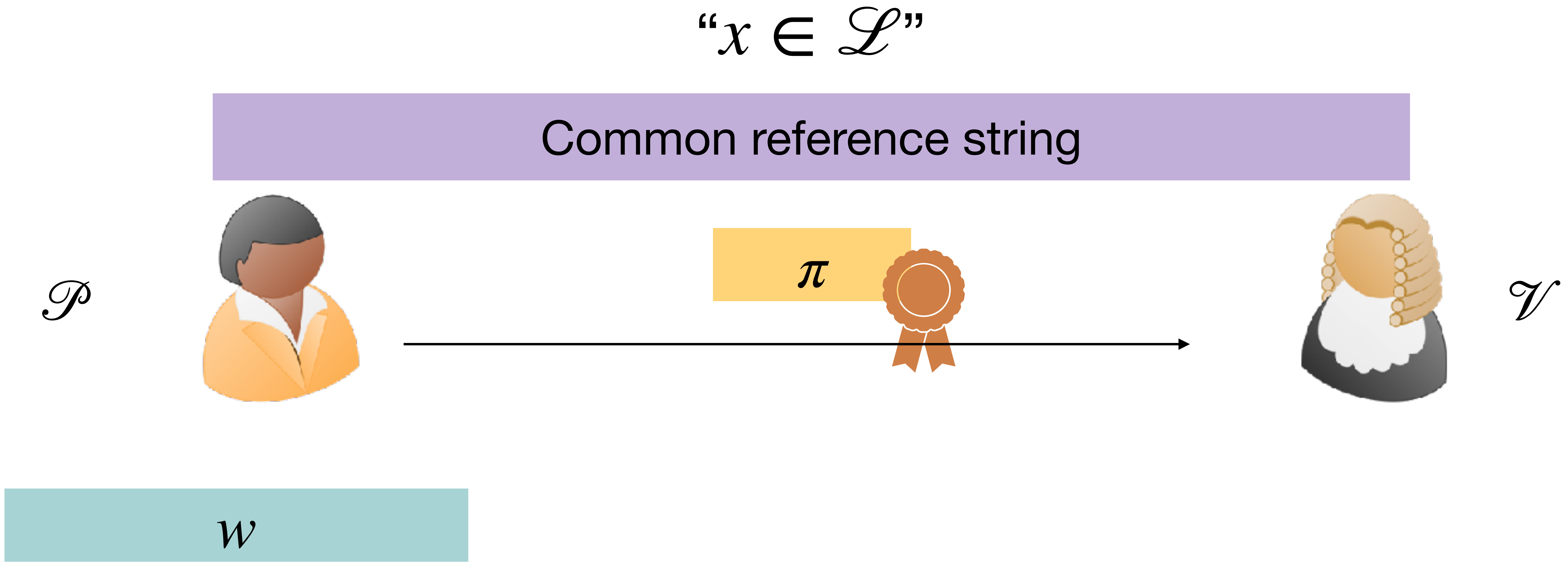


- **Succinctness:** $|\pi| \ll |w|$.

Succinct Non-Interactive Arguments for NP



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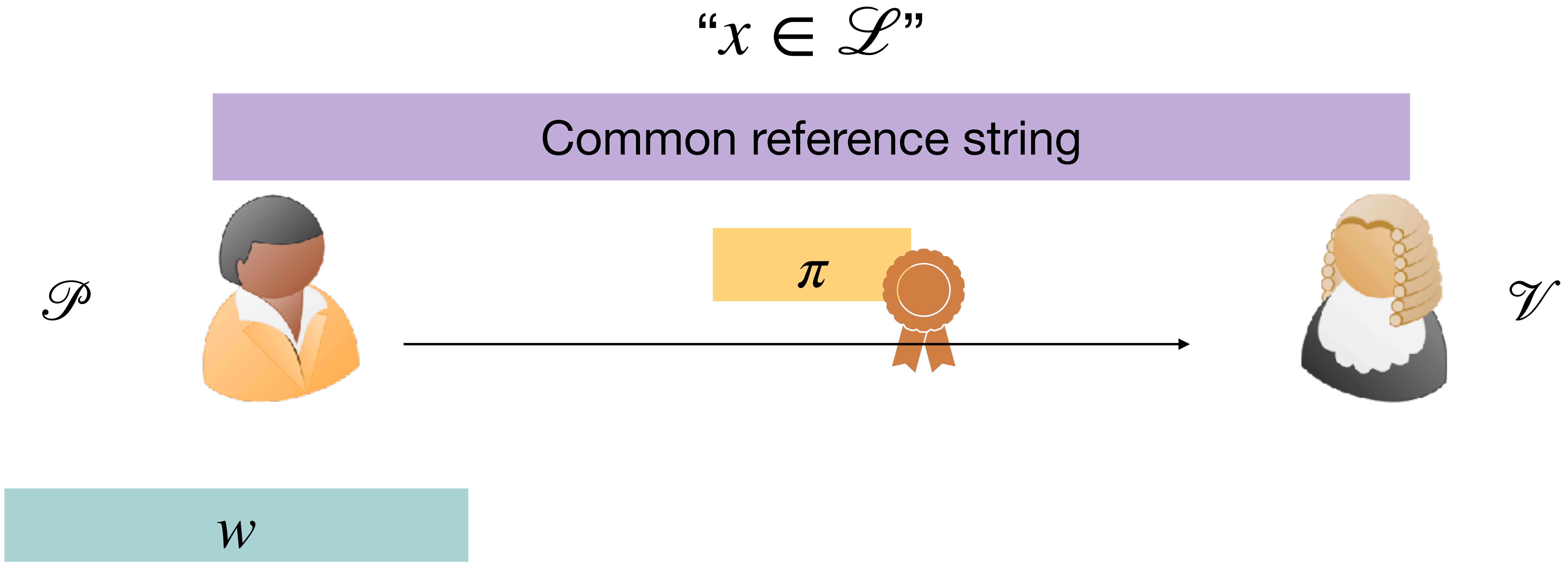

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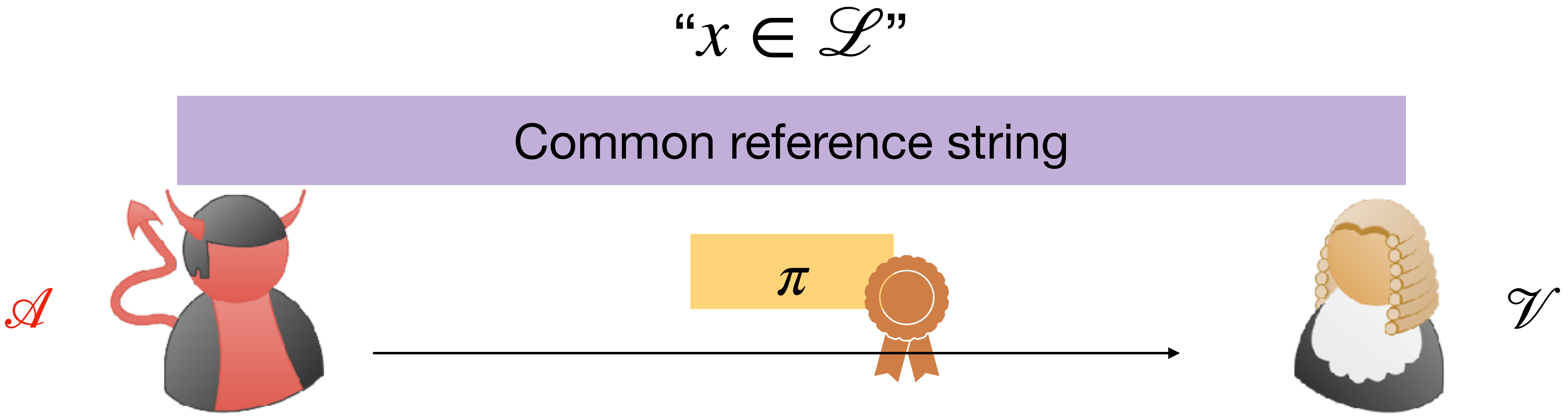

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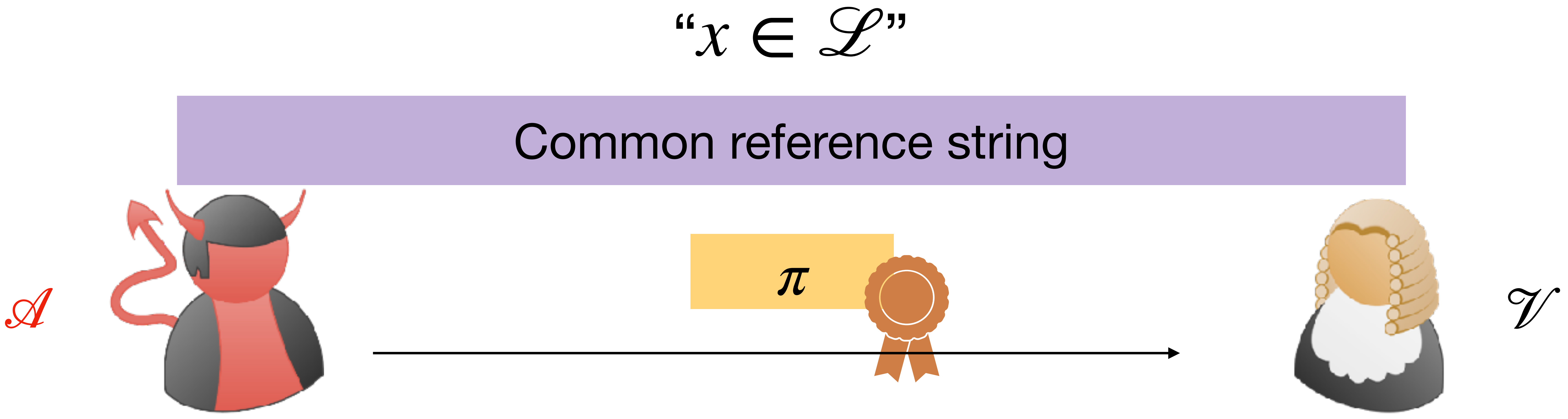

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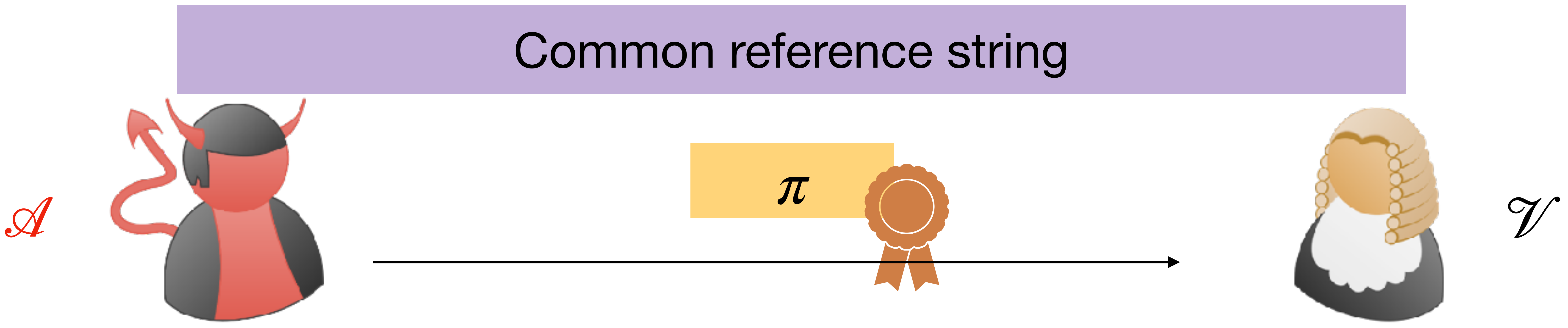

$$\Pr_{\text{crs}}[\pi \leftarrow \mathscr{A}(\text{crs}) \wedge \mathscr{V}(\text{crs}, x, \pi) = 1] \leq 2^{-\lambda}$$

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SNARGs for NP?

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Random Oracle/Knowledge Assumptions

SNARGs for NP in Standard Model?

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SNARGs for NP

SNARGs for NP in Standard Model?

SNARGs for NP

From indistinguishability obfuscation
[SW ’14, WW ’24, **M**PV ’24, WZ ’25, WW ’25]

SNARGs for NP in Standard Model?

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From witness encryption* [JKL**M** ’25]

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Obfustopia

Not known from standard, post-quantum assumptions

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Natural subclasses of NP ∩ coNP [JKLV24, JKL**M**25]

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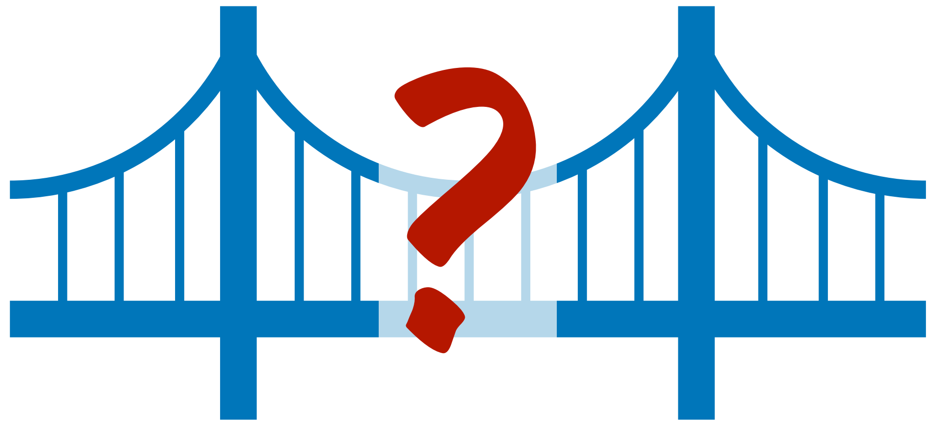

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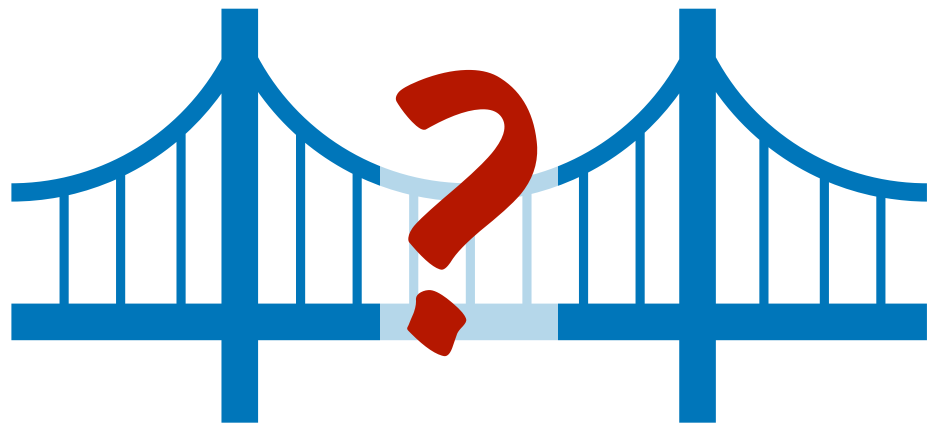

Can we build SNARGs from LWE/Bilinear Maps/etc?

From Learning with Error, Bilinear Maps, Etc.

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How to construct SNARGs?

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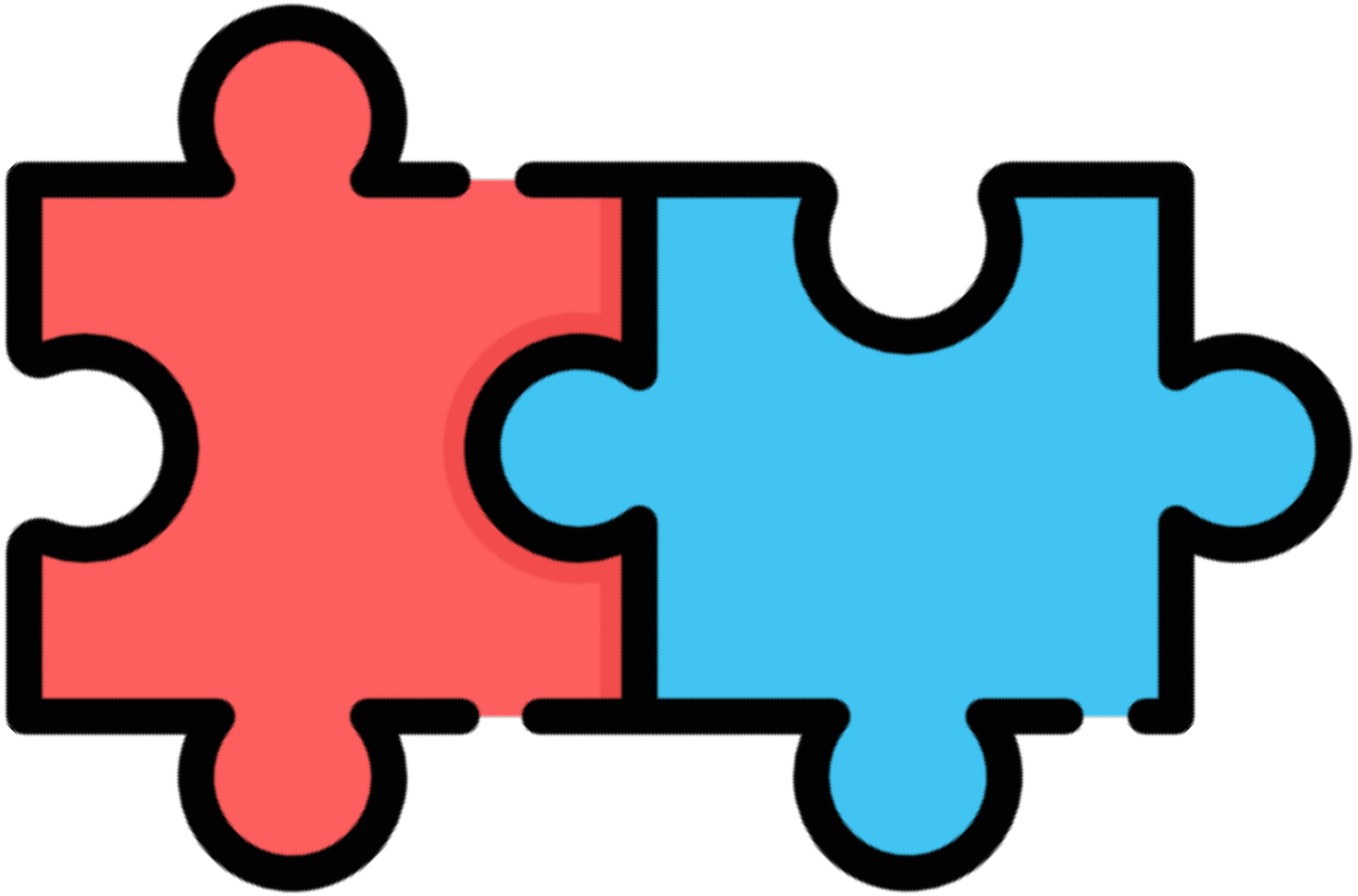

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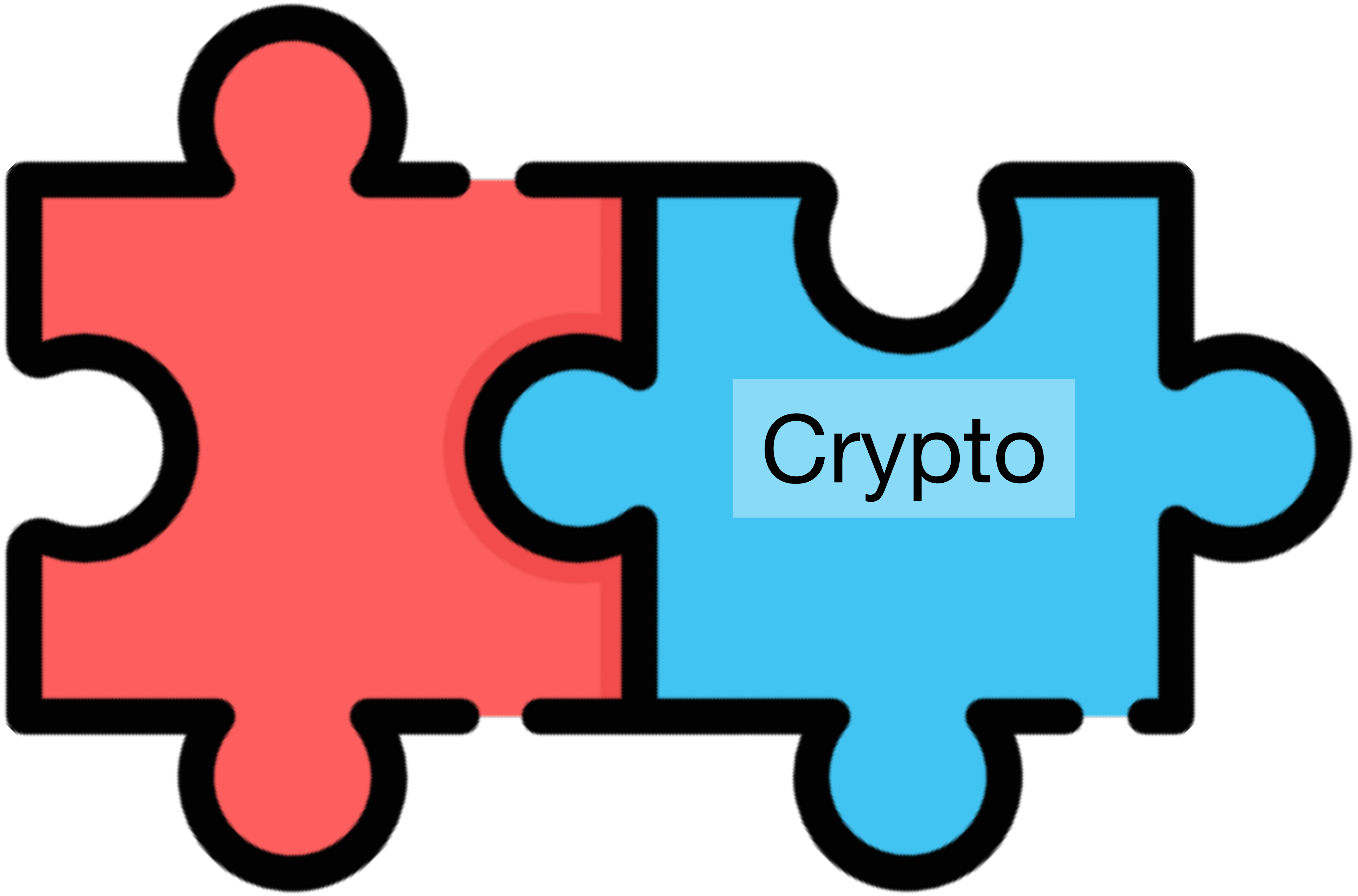

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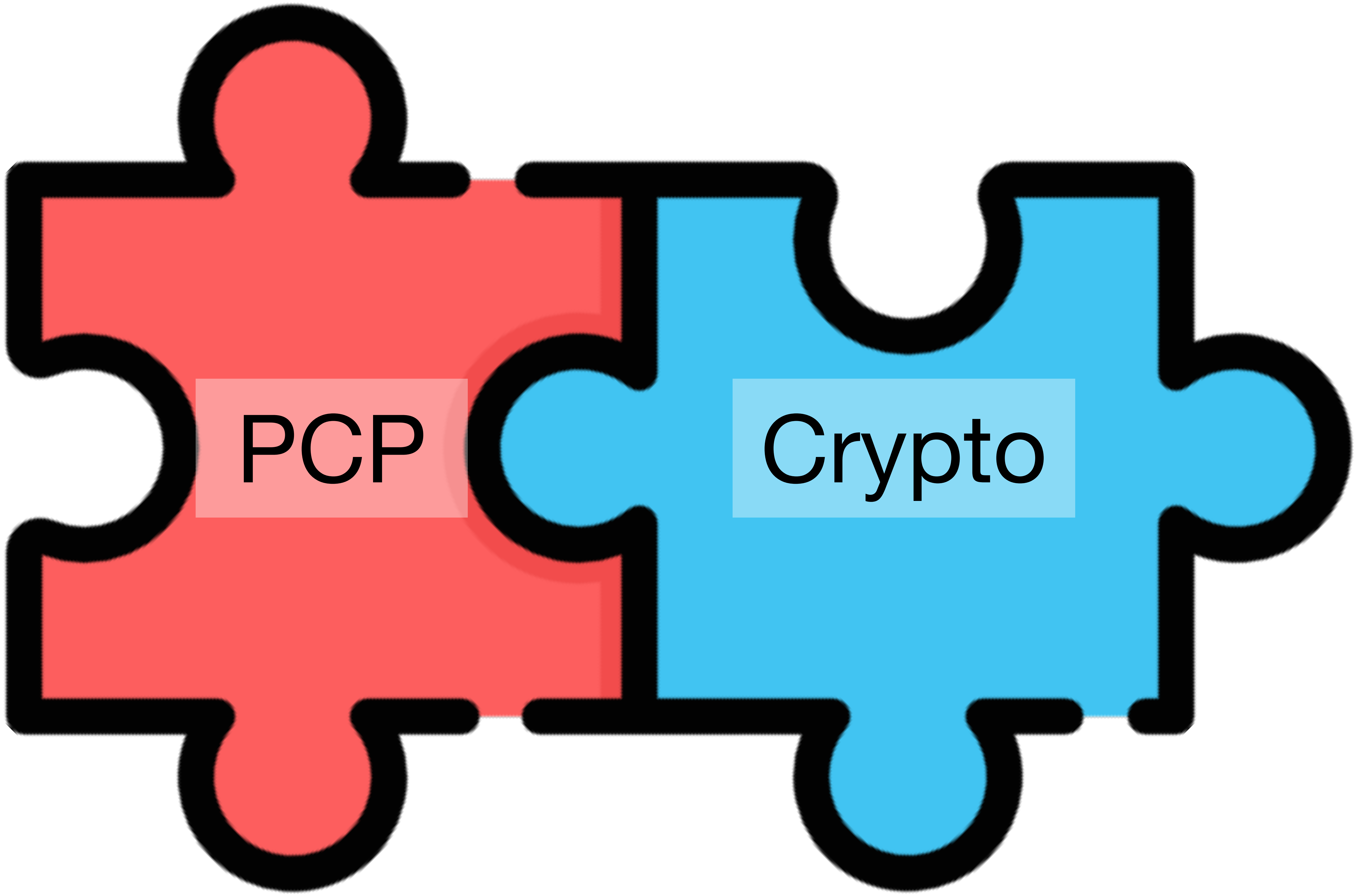

Probabilistically Checkable Proofs

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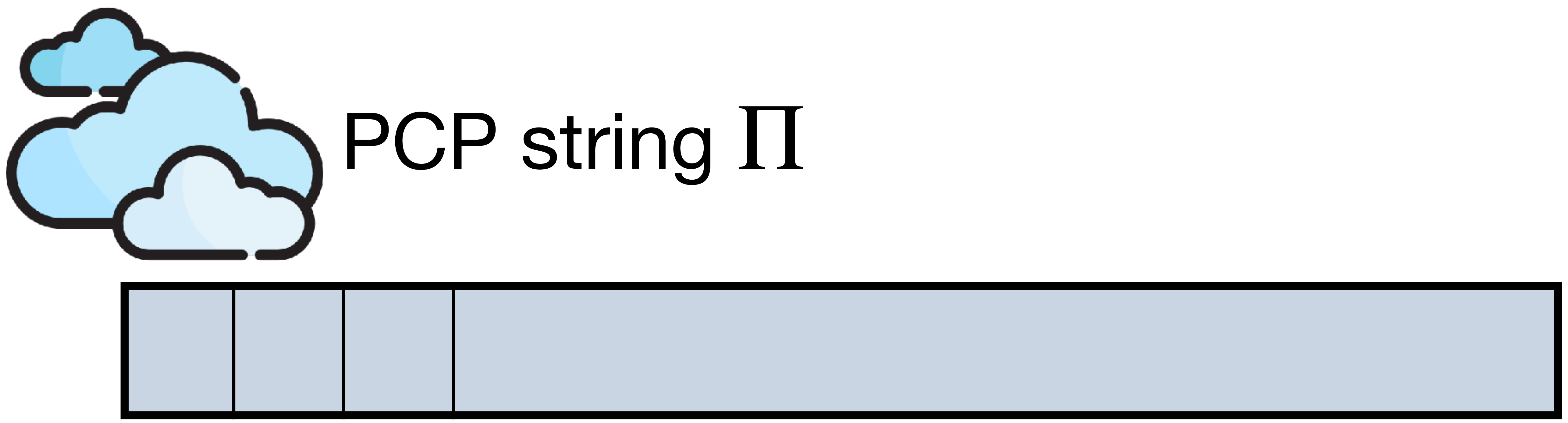
PCP string Π

Probabilistically Checkable Proofs



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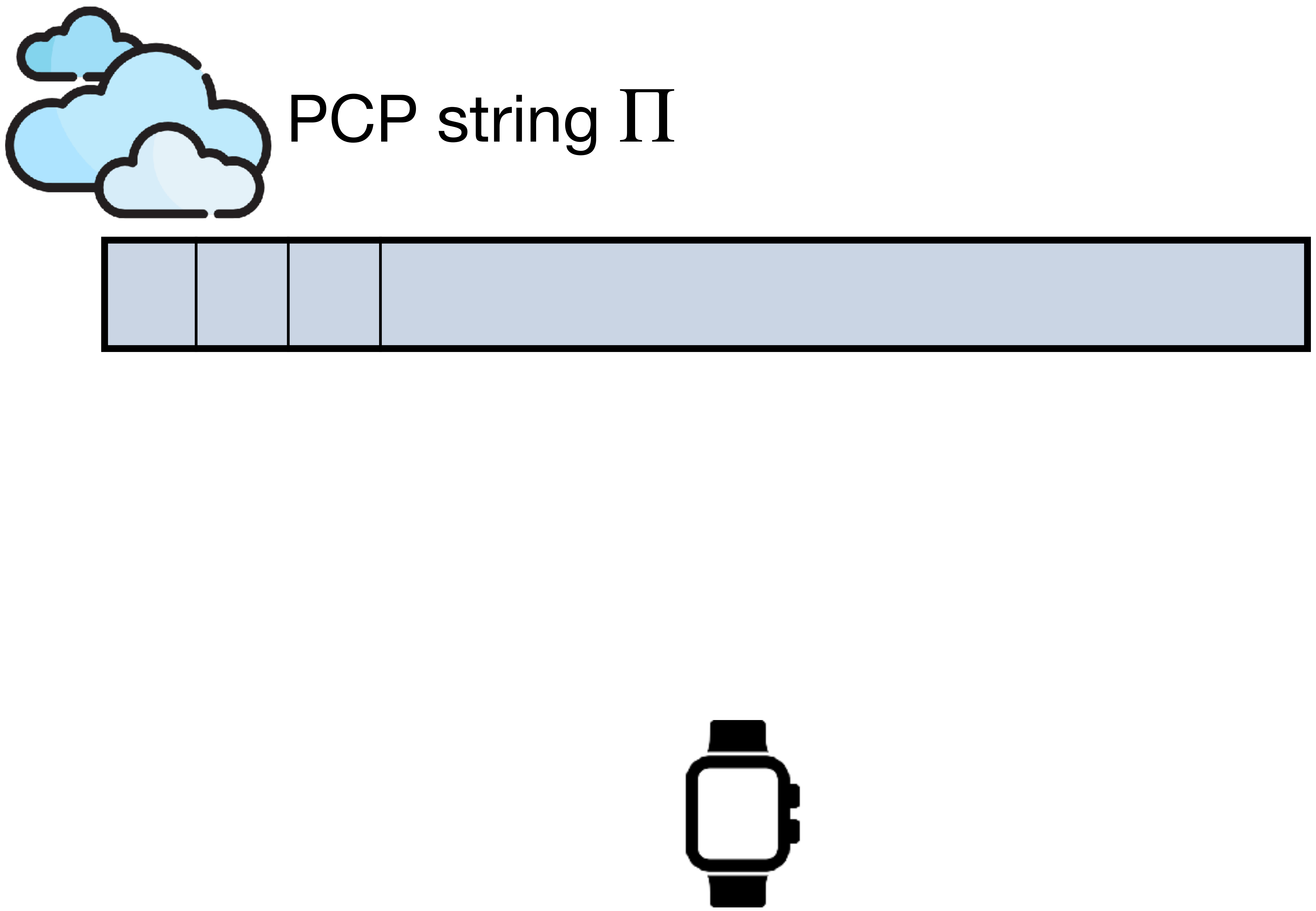

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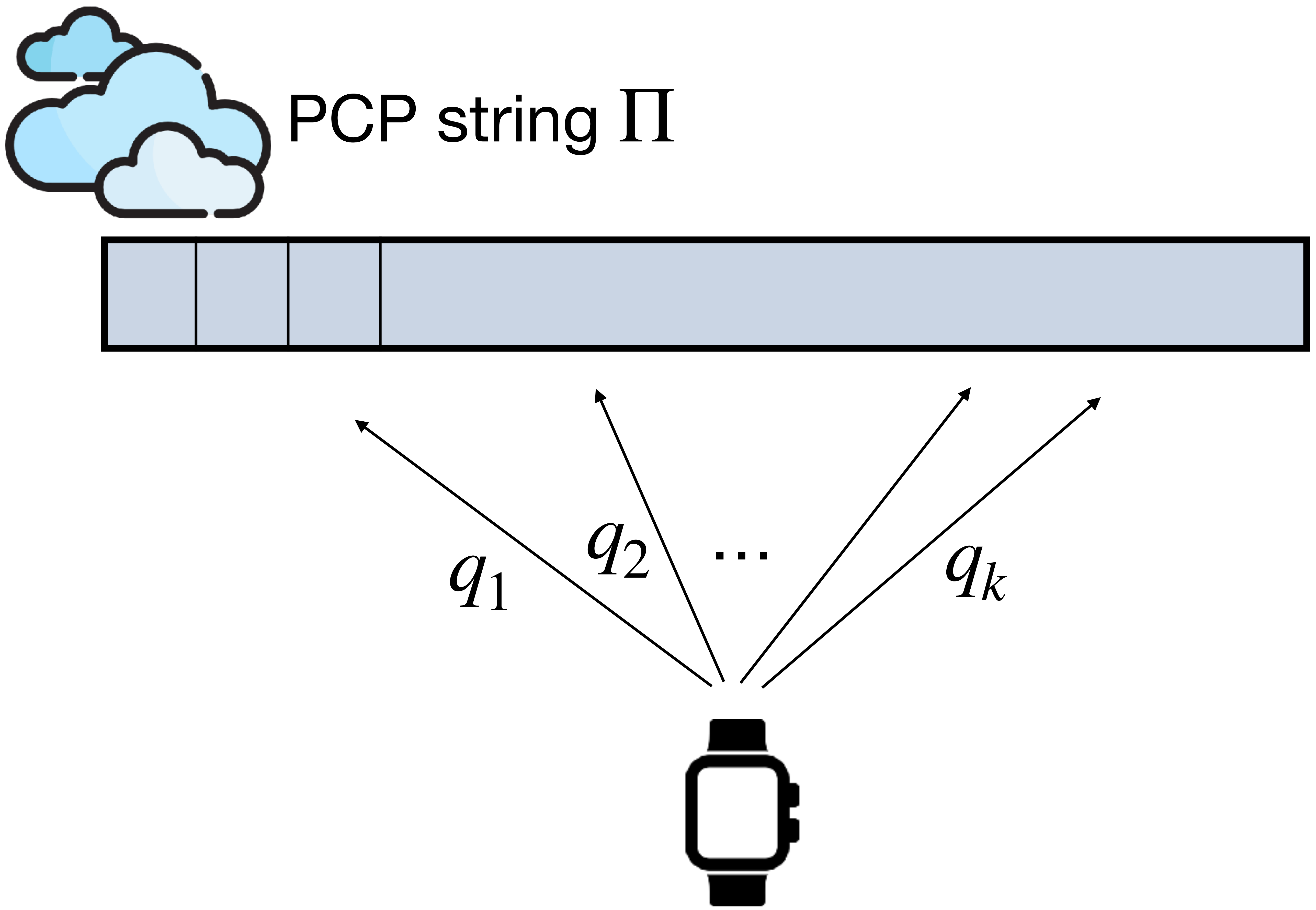

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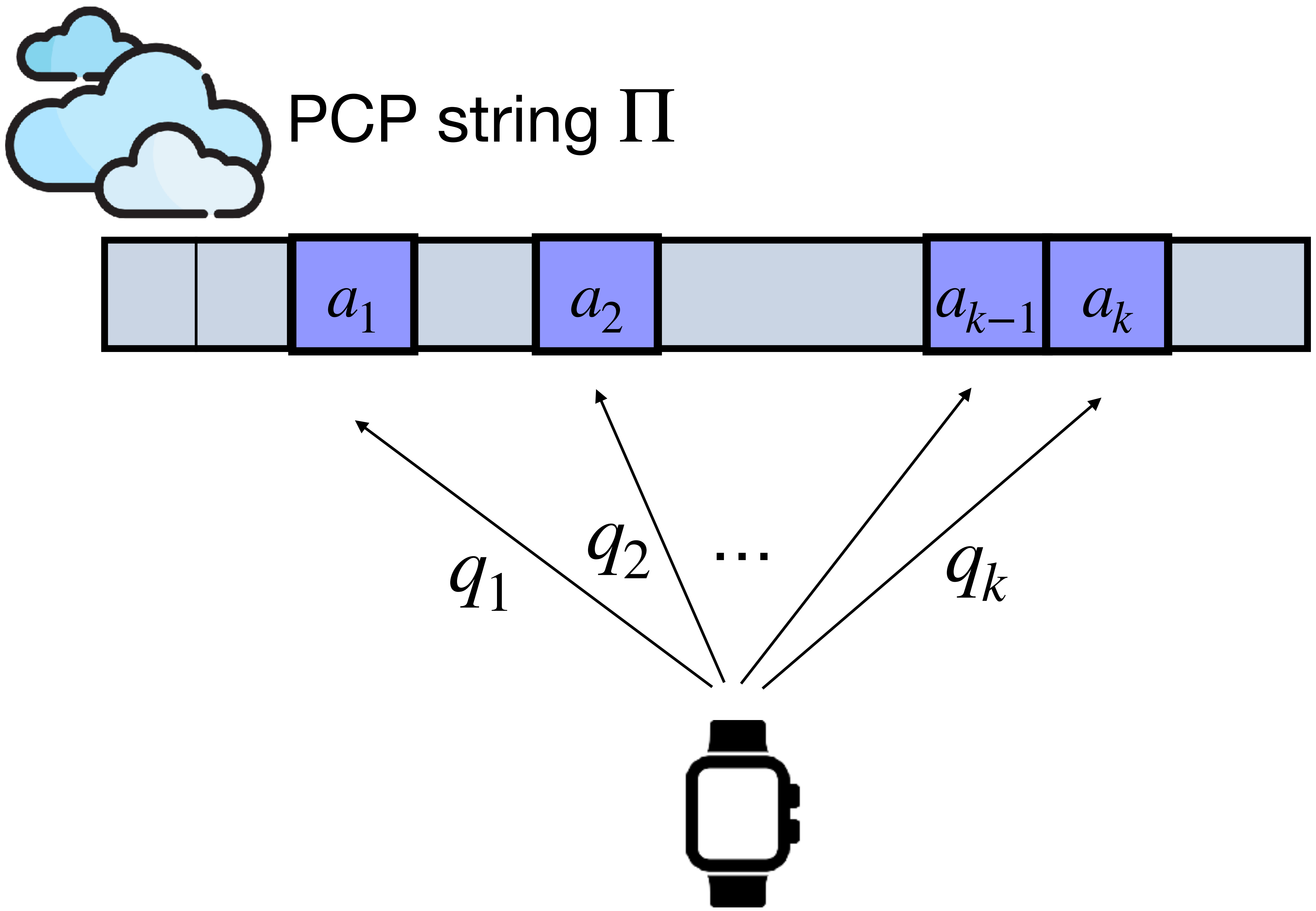

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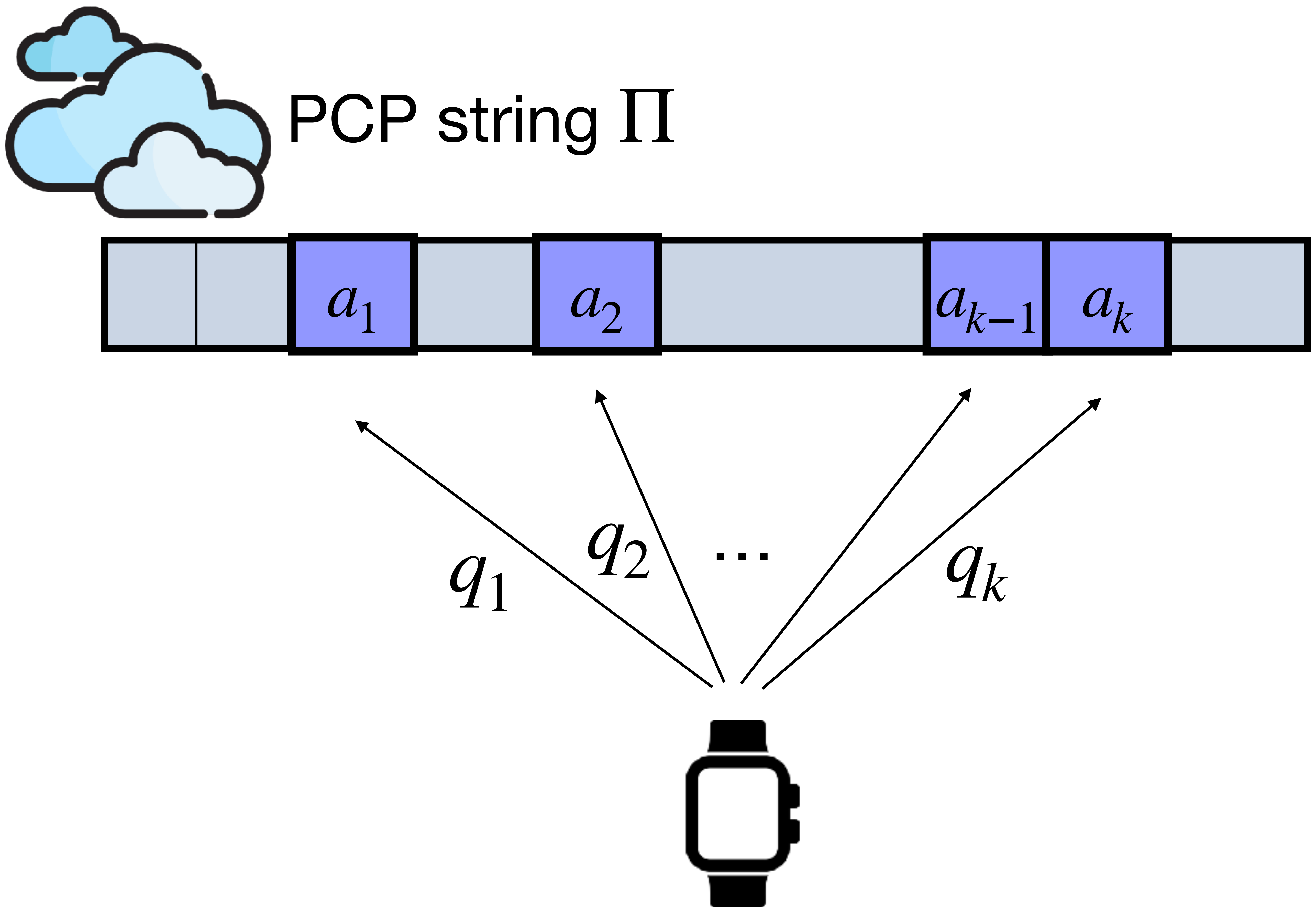
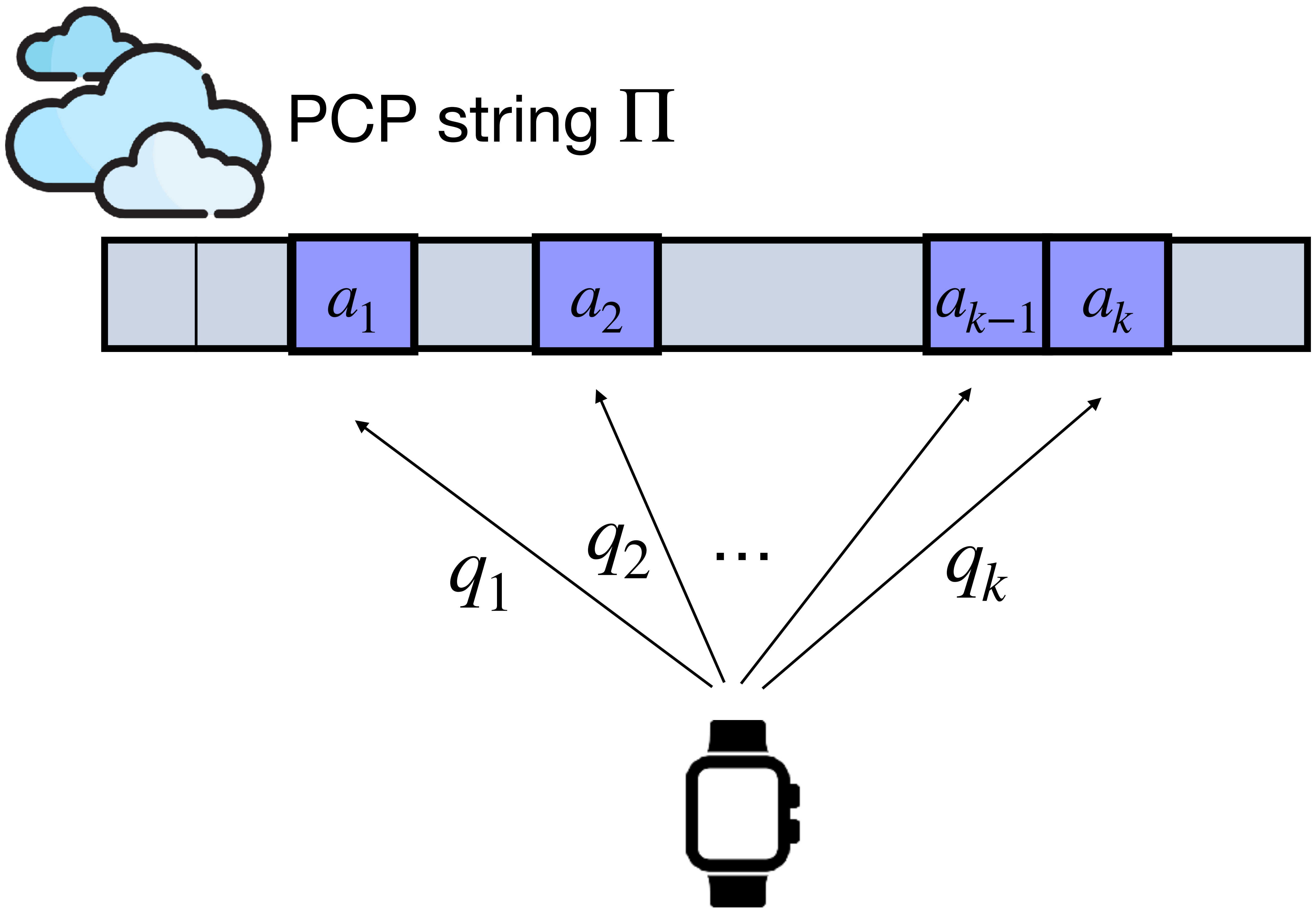


- **Locality:** $k \ll |w|$

Probabilistically Checkable Proofs



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- **Correctness:** For honest Π, V always accepts.

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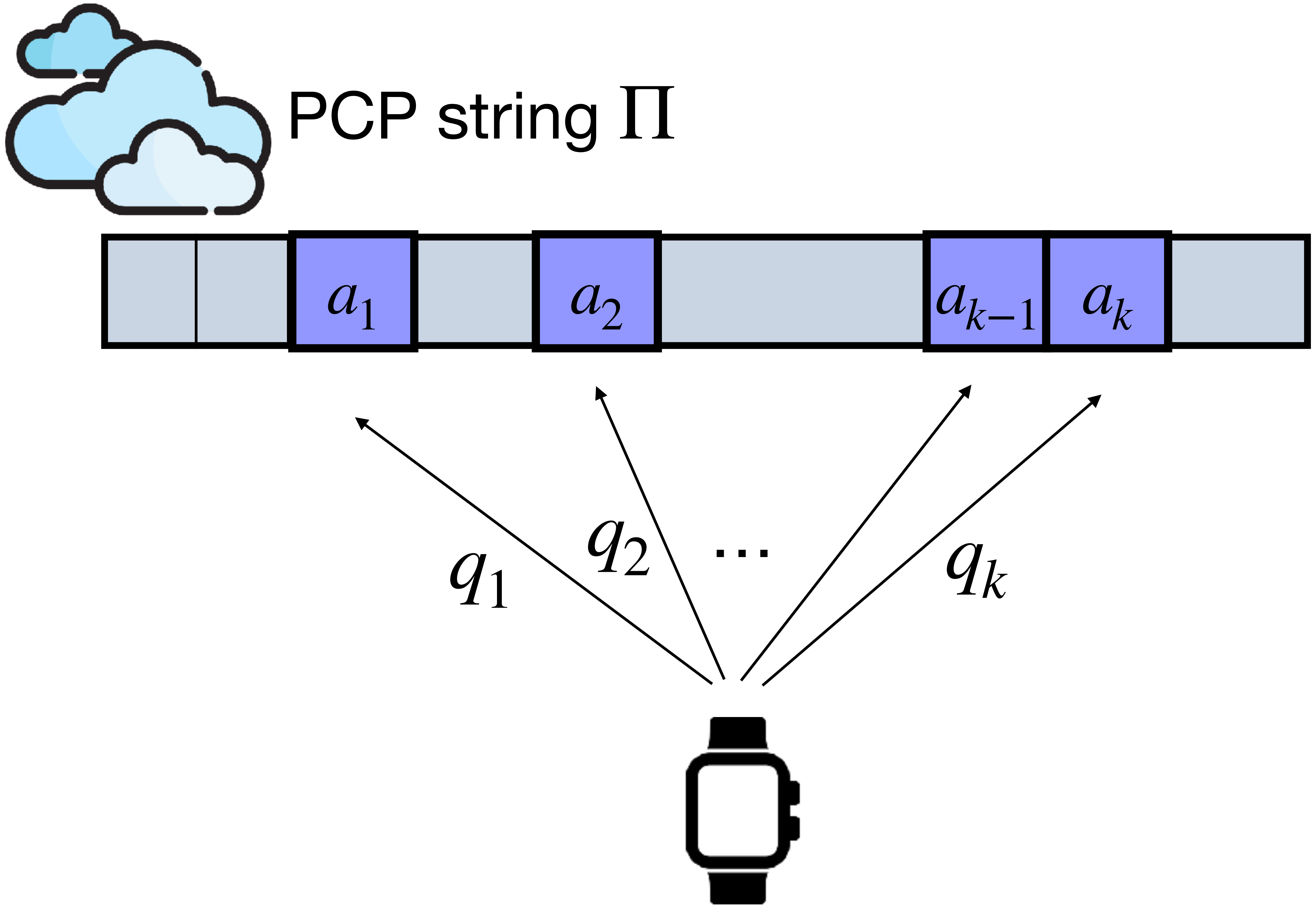

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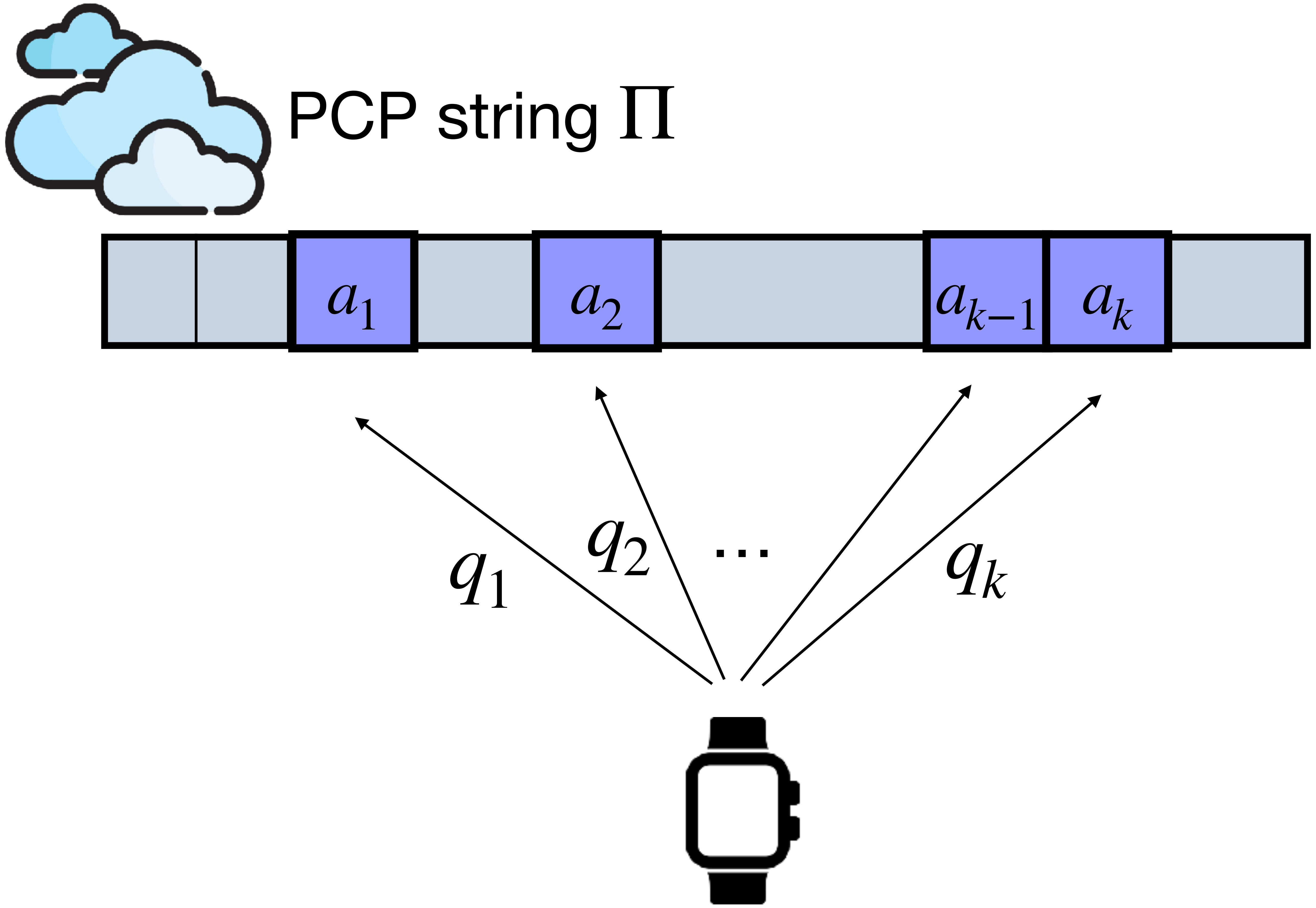

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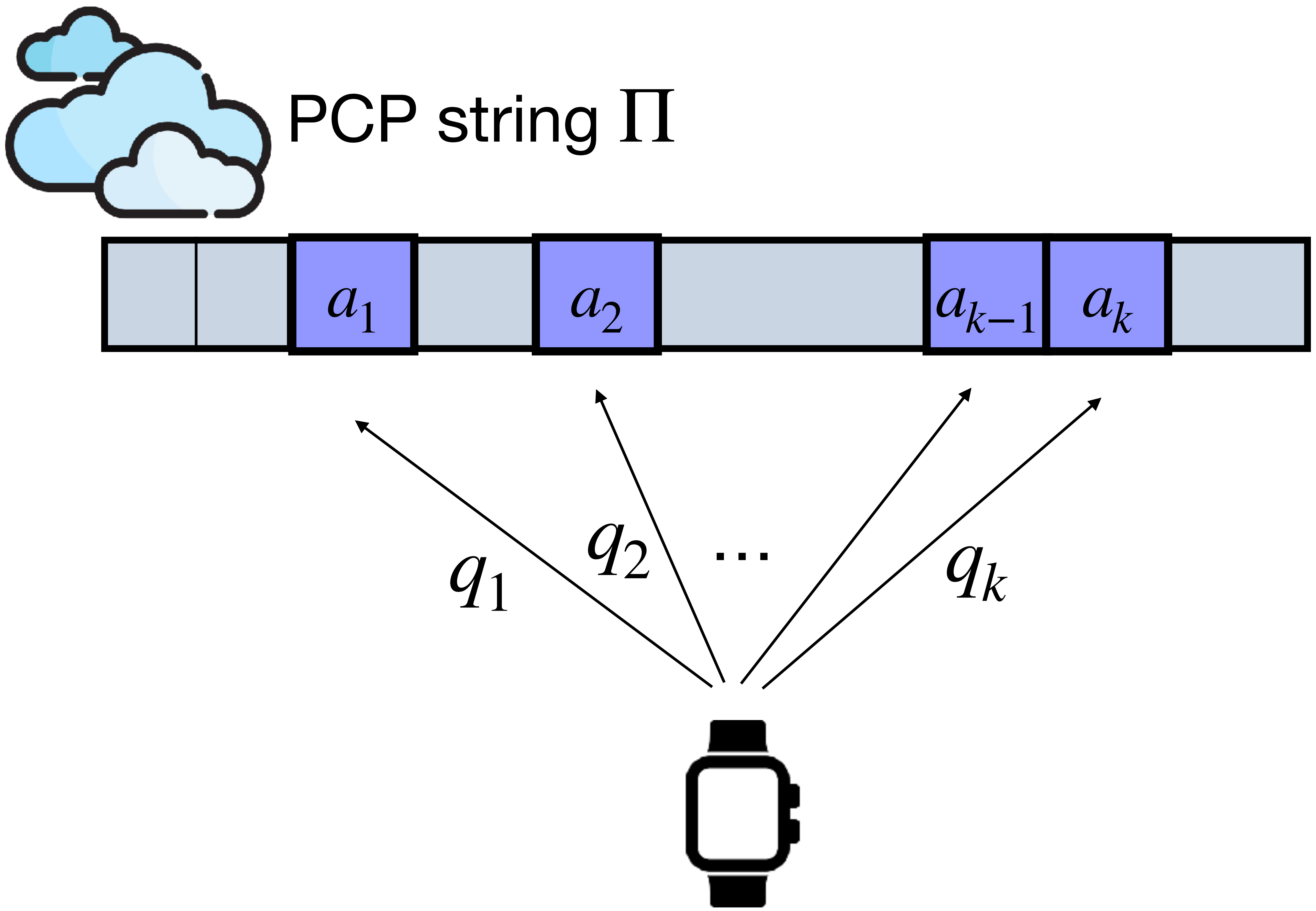
Theorem [ALMSS '92]. There exists a PCP of length poly(n) and locality polylog(n)

PCP to SNARG?



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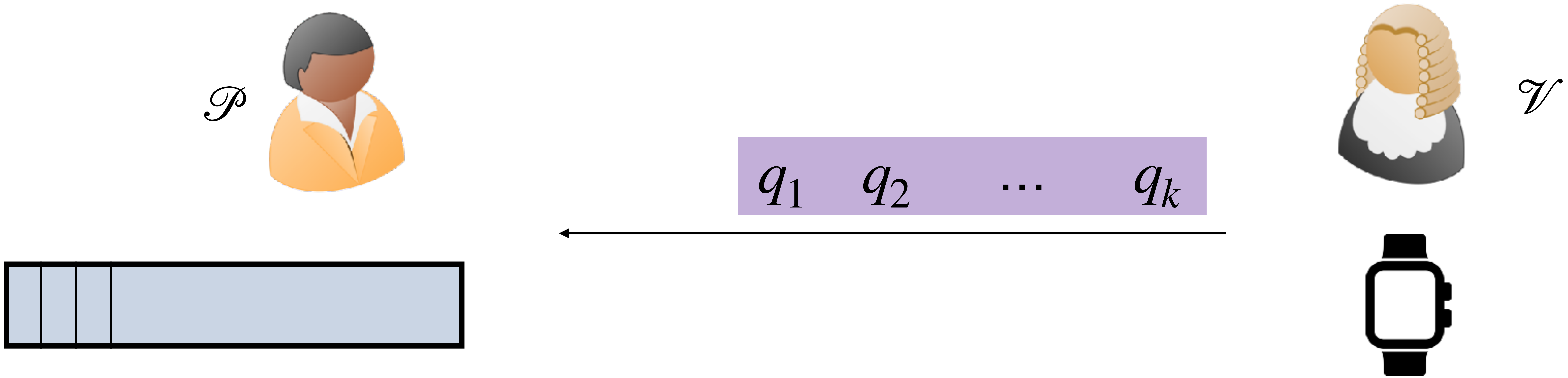

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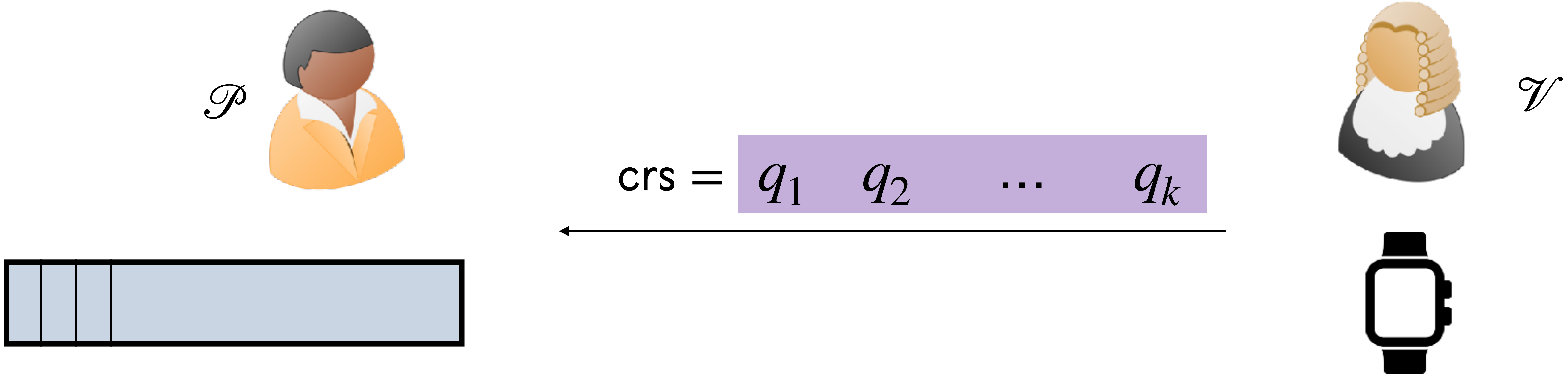

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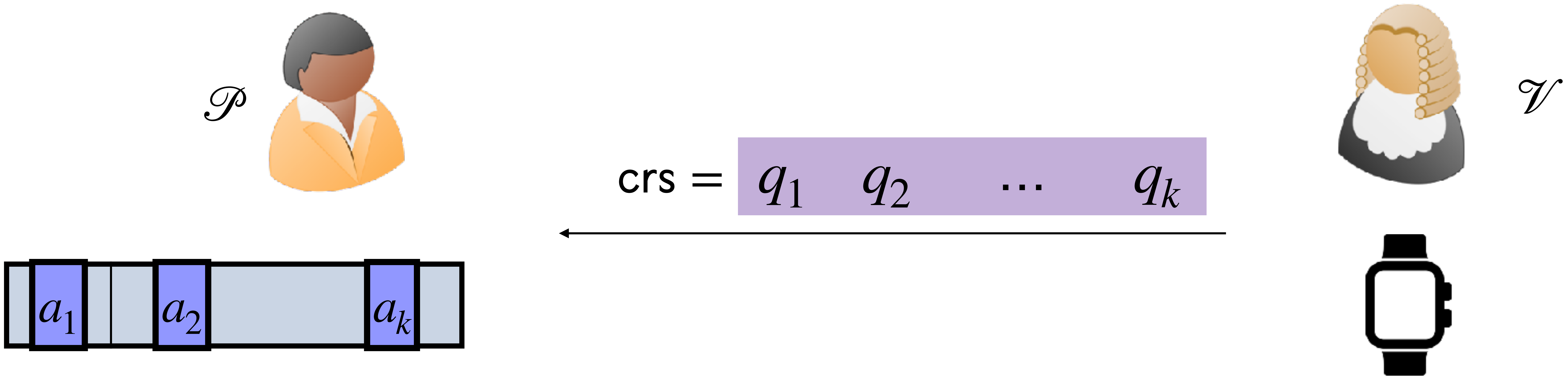
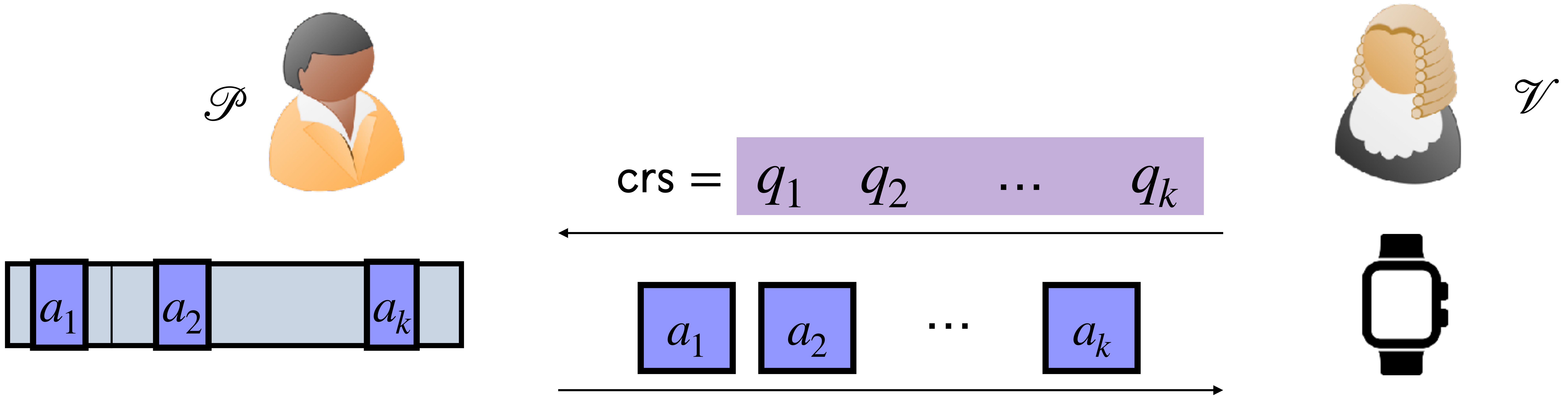


$\mathscr{P}$
$\mathscr{V}$
$\text{crs} = q_1 \quad q_2 \quad \cdots \quad q_k$
a_1
a_2
a_k

PCP to SNARG?



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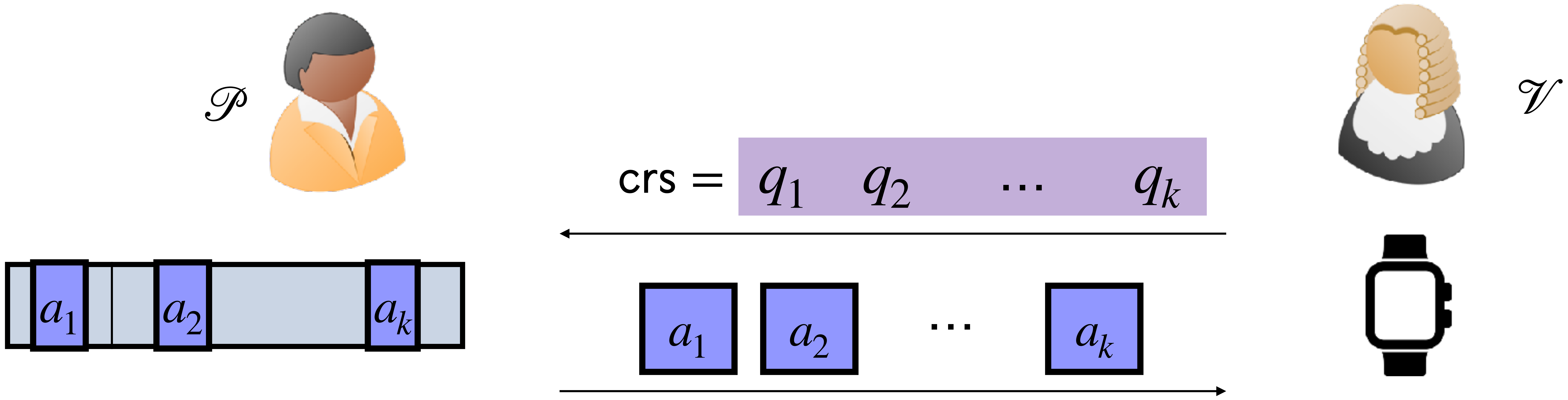
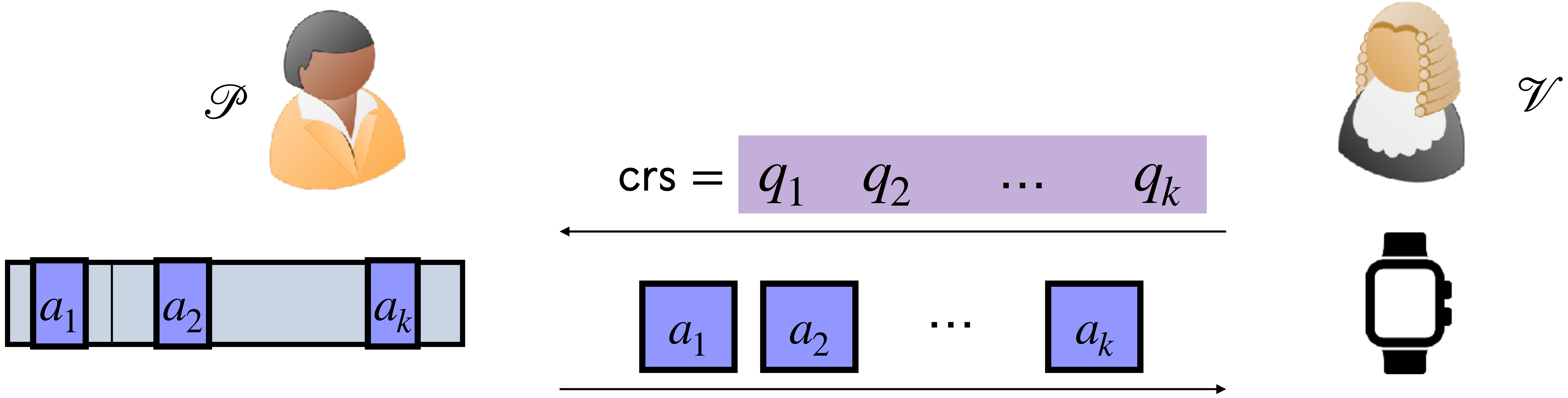


✓ Succinct and Correct

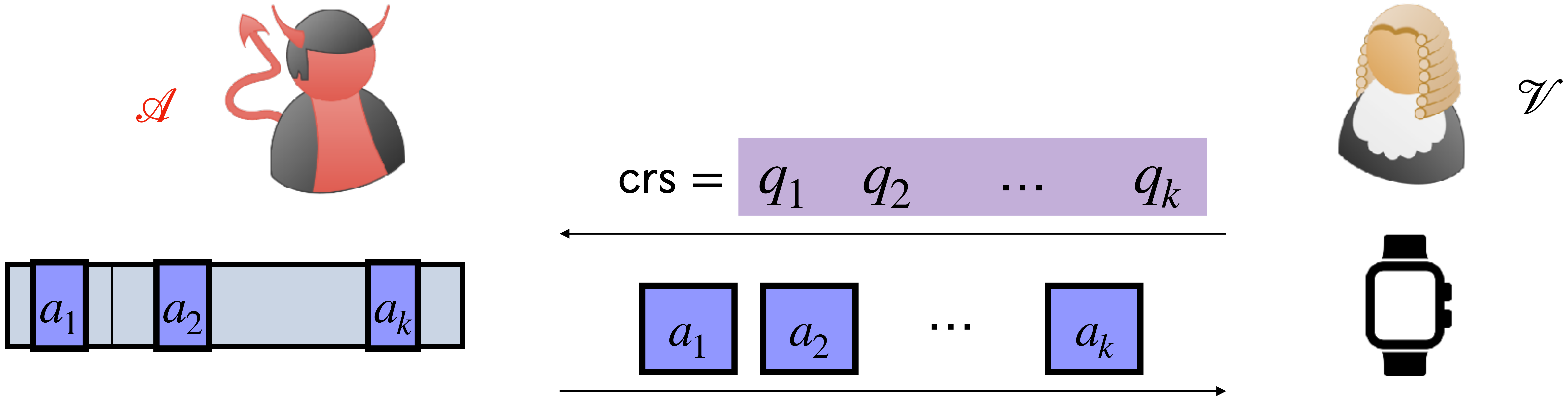
PCP to SNARG?



✓ Succinct and Correct

✗ **Not sound**

PCP to SNARG?



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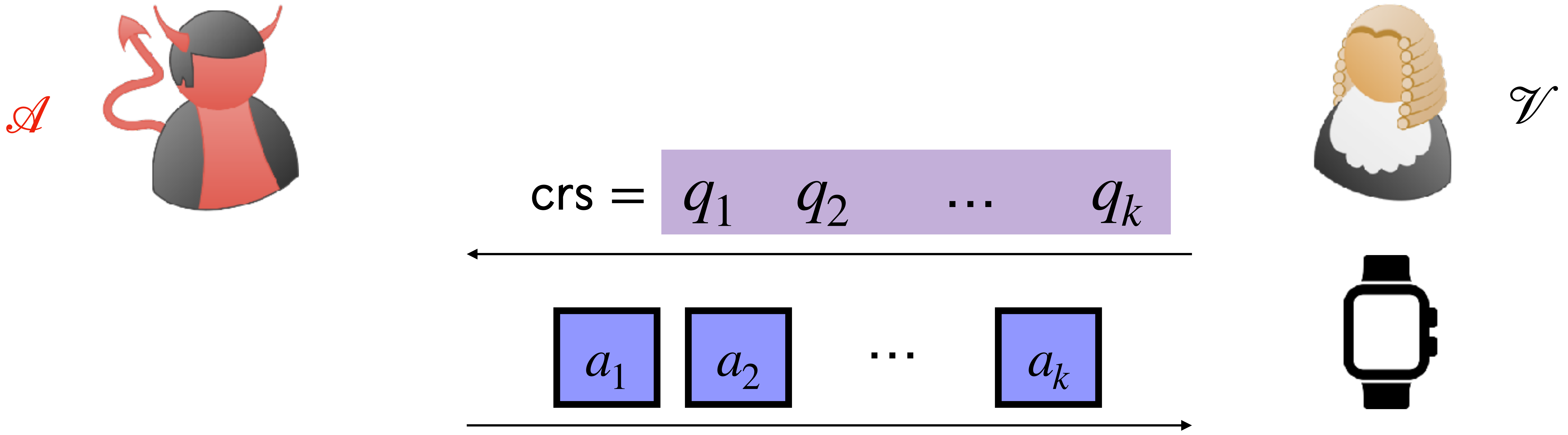

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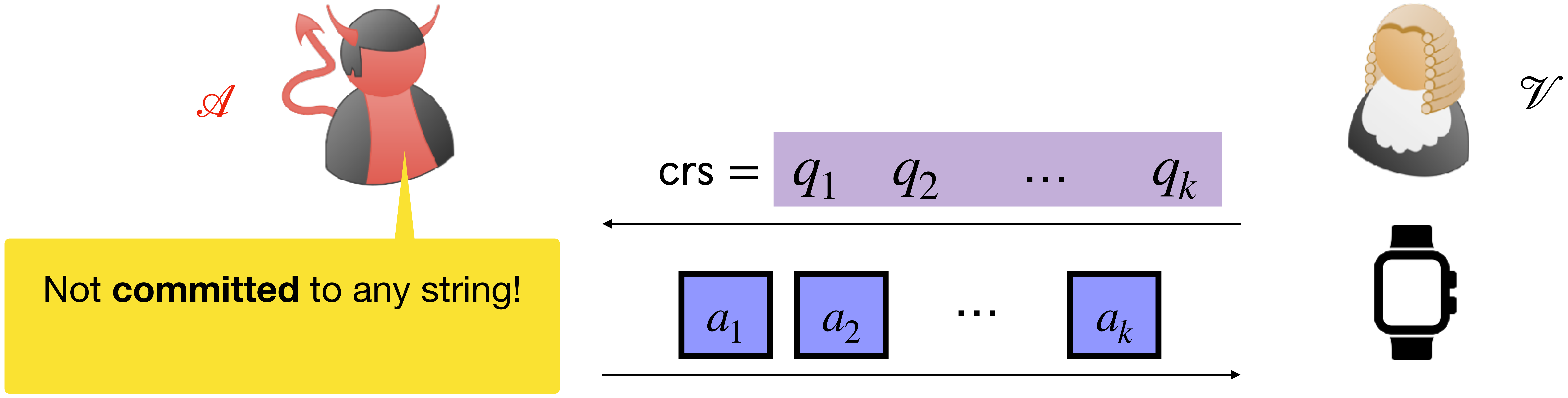

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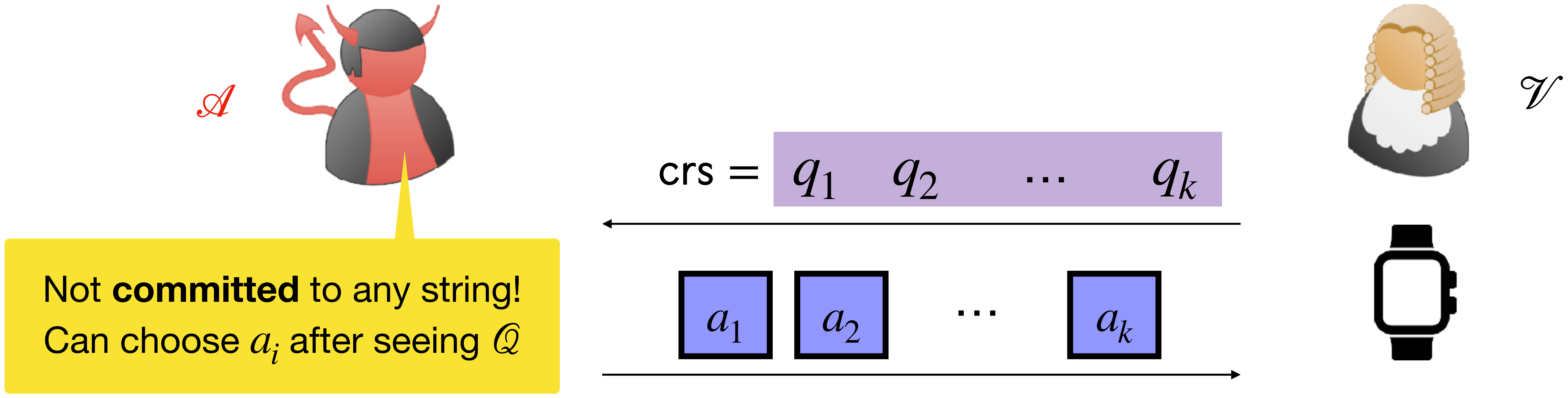

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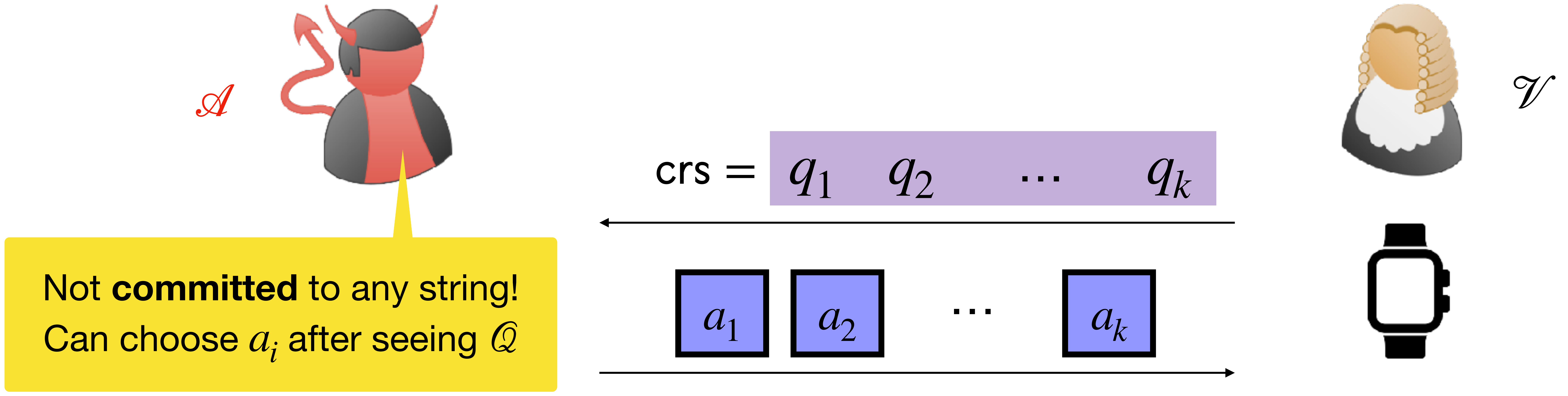

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Need some cryptography in this compiler to restrict $\mathscr{A}$!

PCP to SNARG?

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- **Recipe #1:** [Kilian ’92, Micali ’94] (not in talk)

PCP to SNARG?

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PCP +  Commitments

PCP to SNARG?

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PCP +  Commitments = **Interactive** Argument

PCP to SNARG?

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PCP +  Commitments

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PCP +  Commitments + Fiat-Shamir

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PCP to SNARG?

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PCP + 💍 Commitments + Fiat-Shamir

- **Recipe #2:** [BMW ’98, KRR ’14]

PCP to SNARG?

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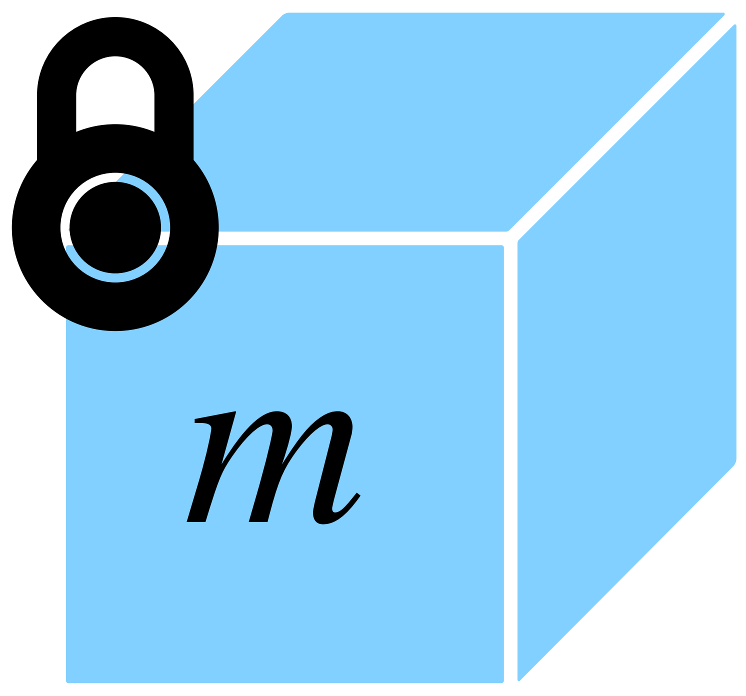
Fully Homomorphic Encryption

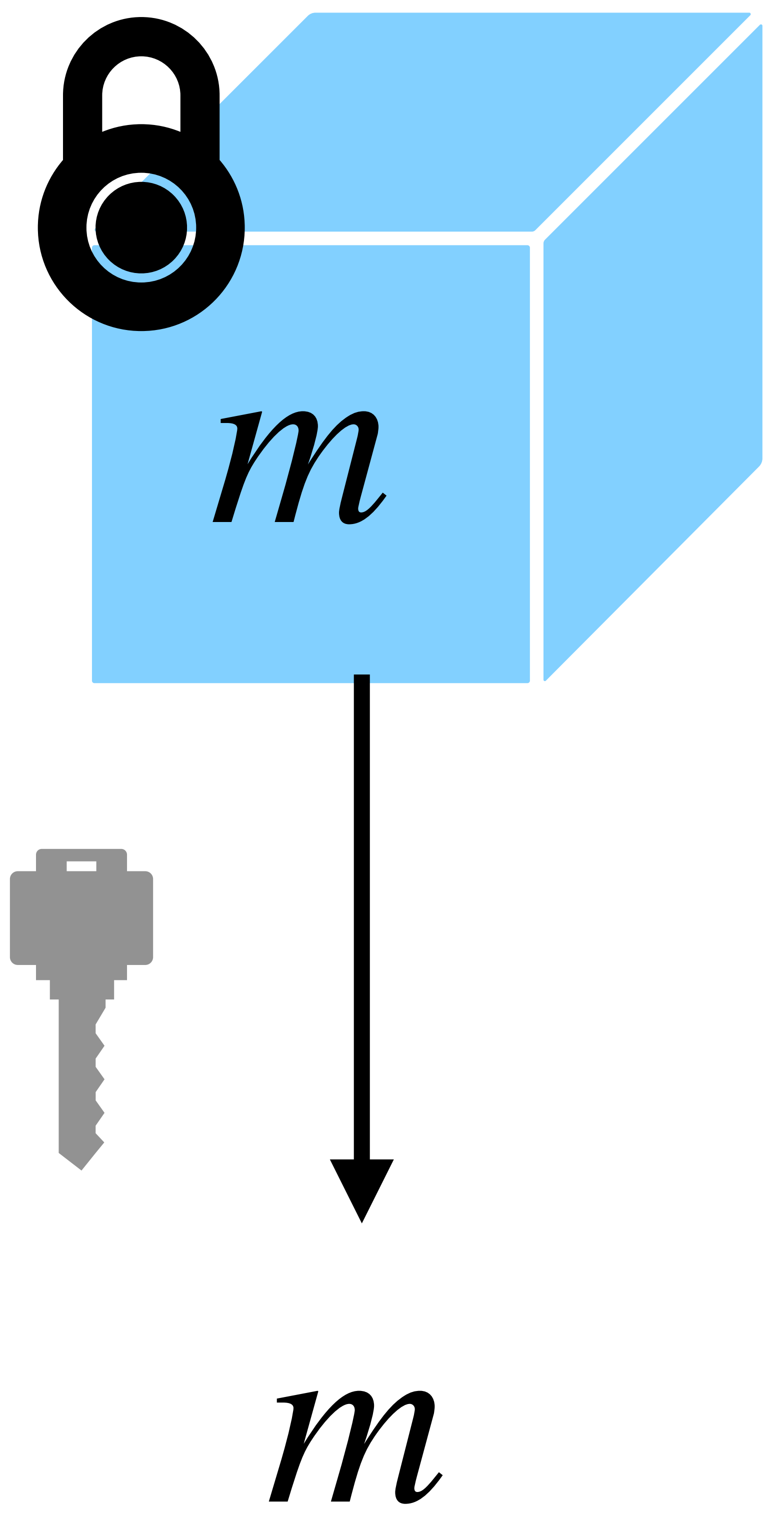
Fully Homomorphic Encryption

m

Fully Homomorphic Encryption



Fully Homomorphic Encryption



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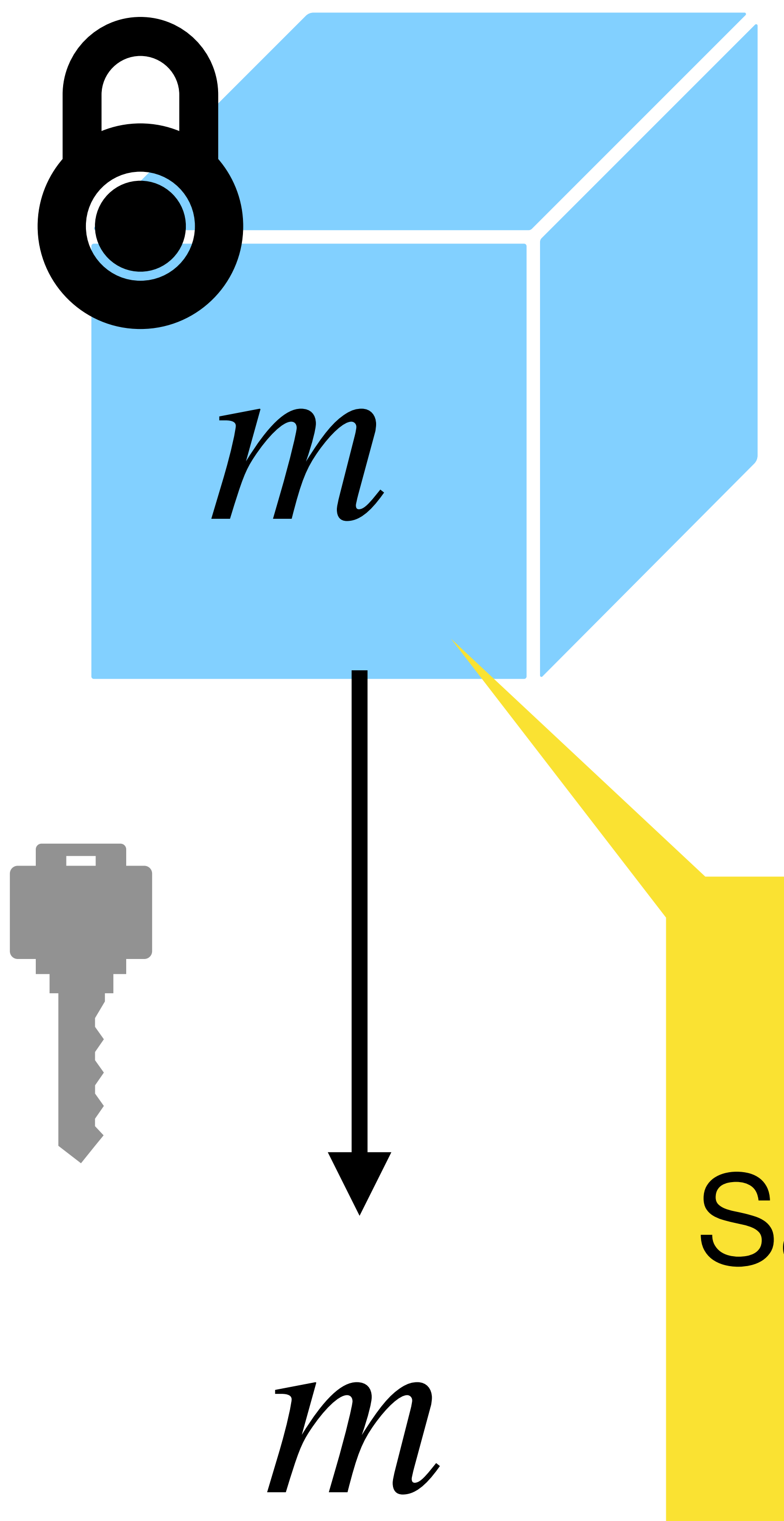
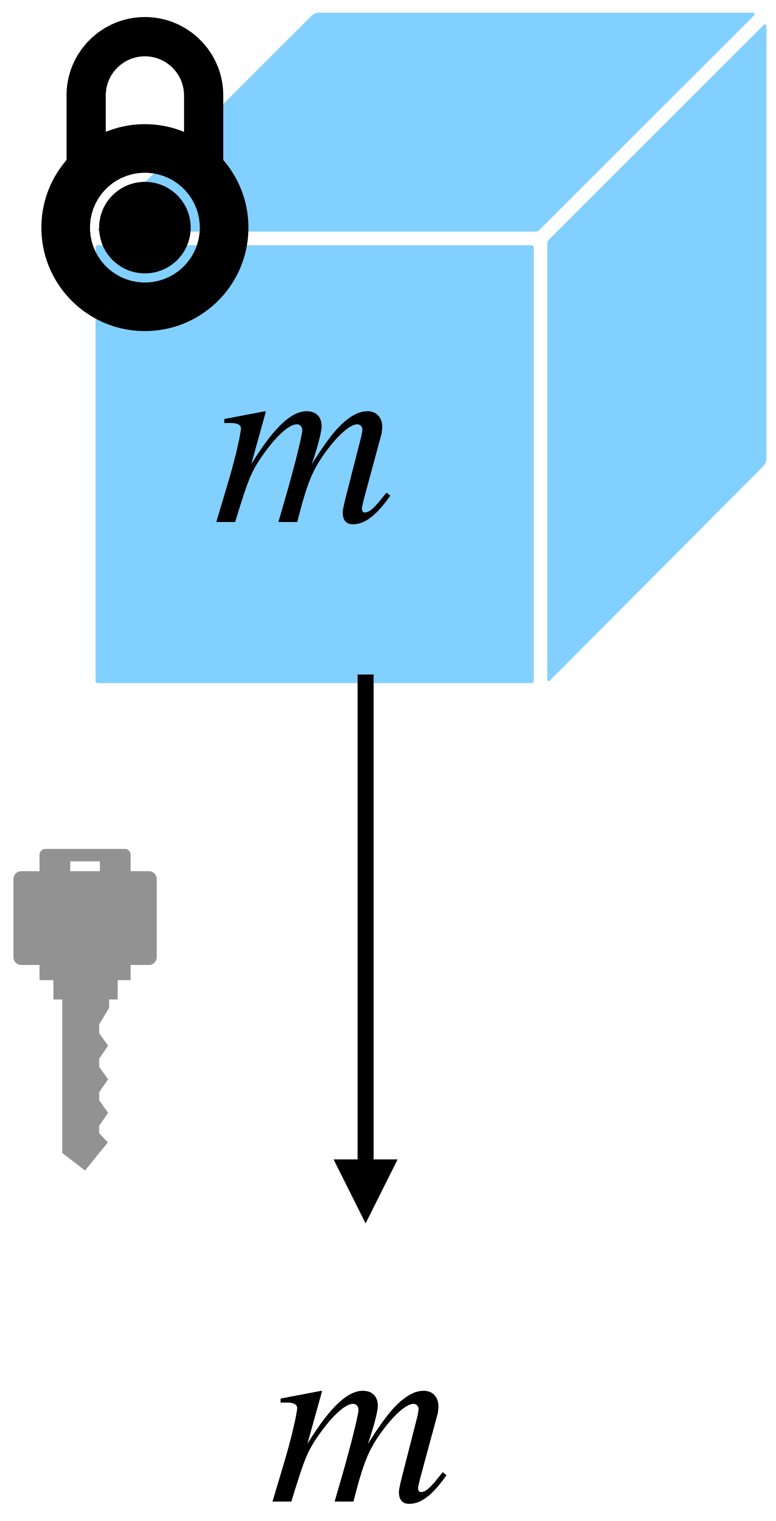


Satisfies usual properties of an encryption scheme

Fully Homomorphic Encryption



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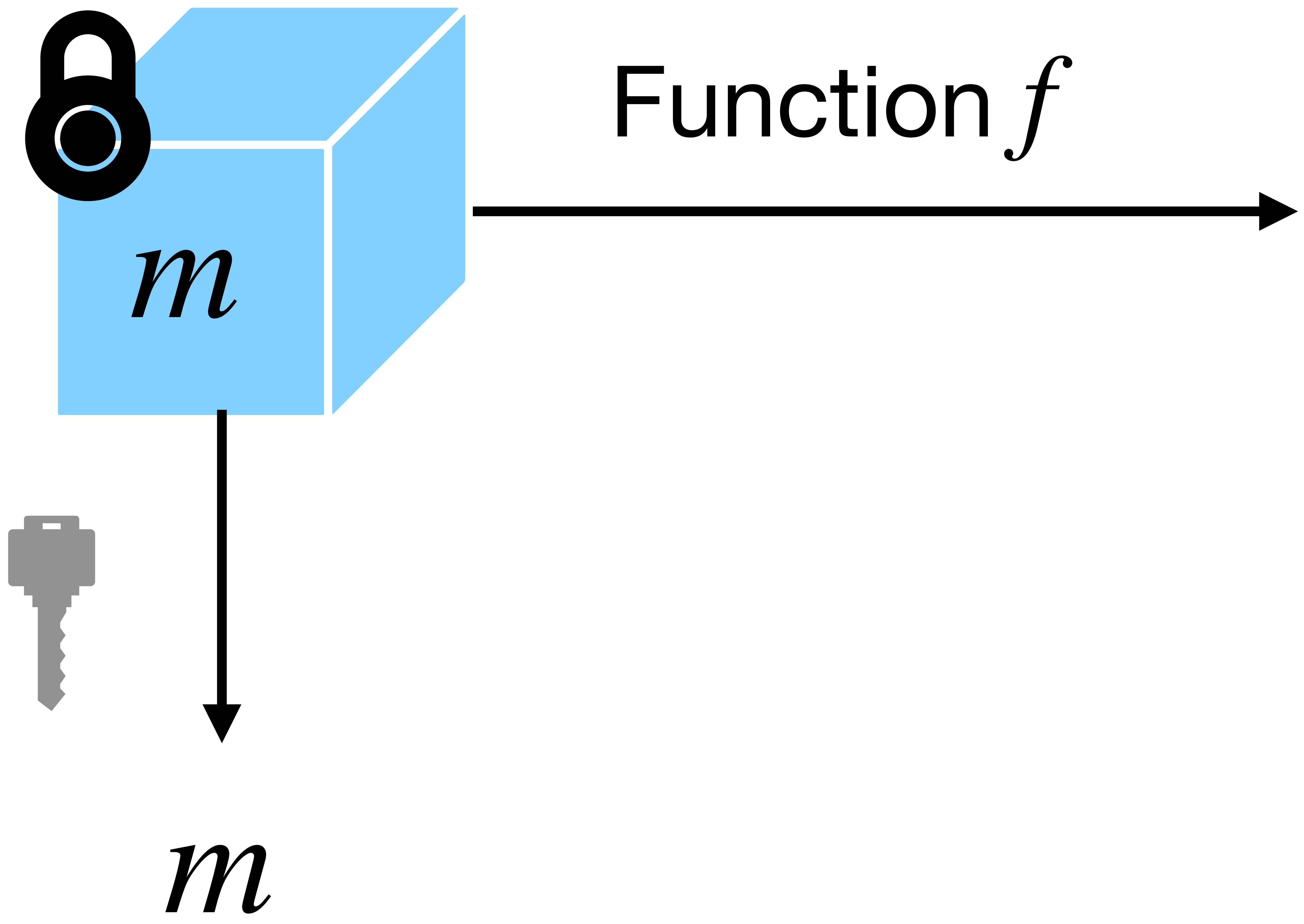

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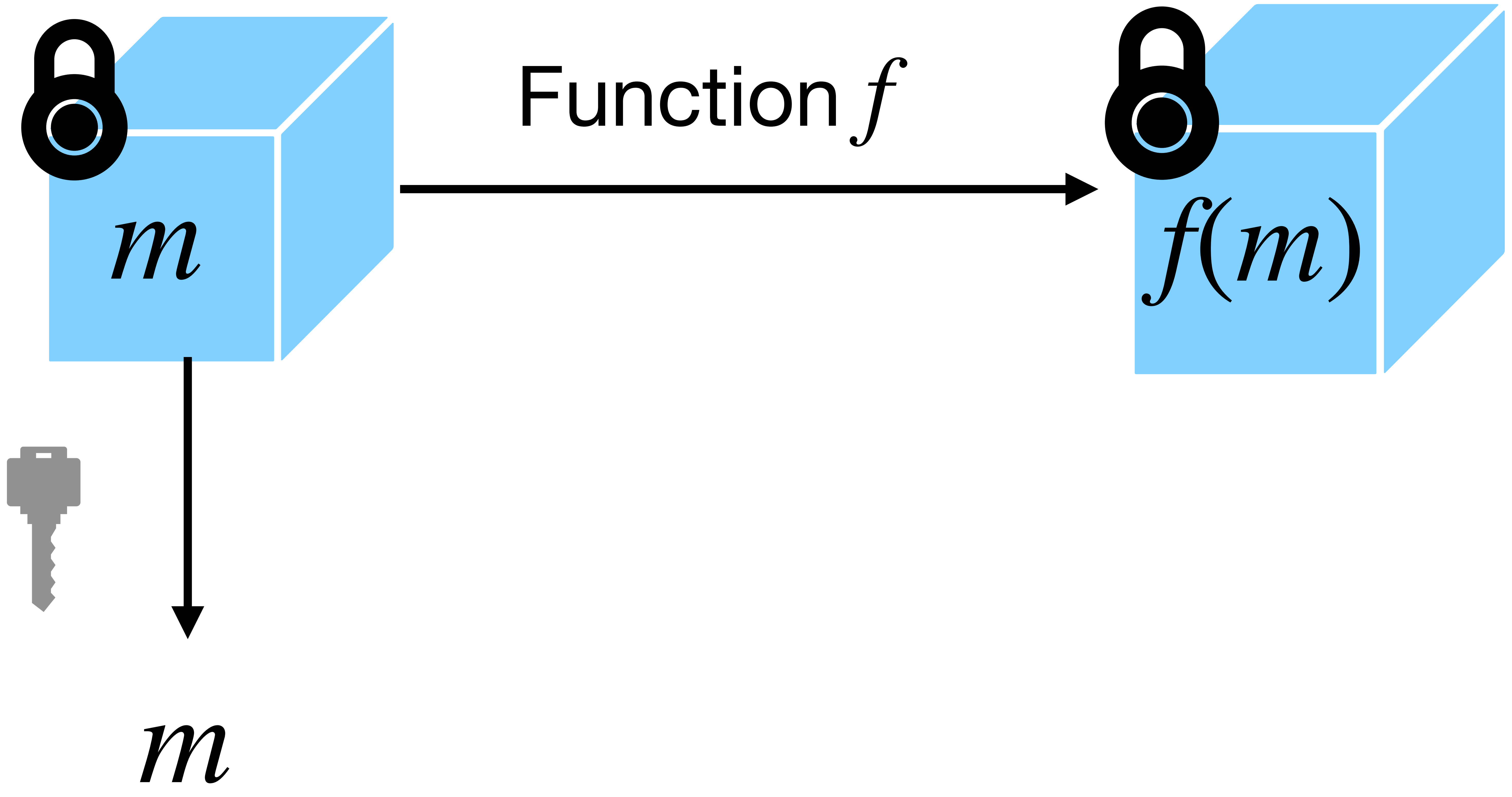

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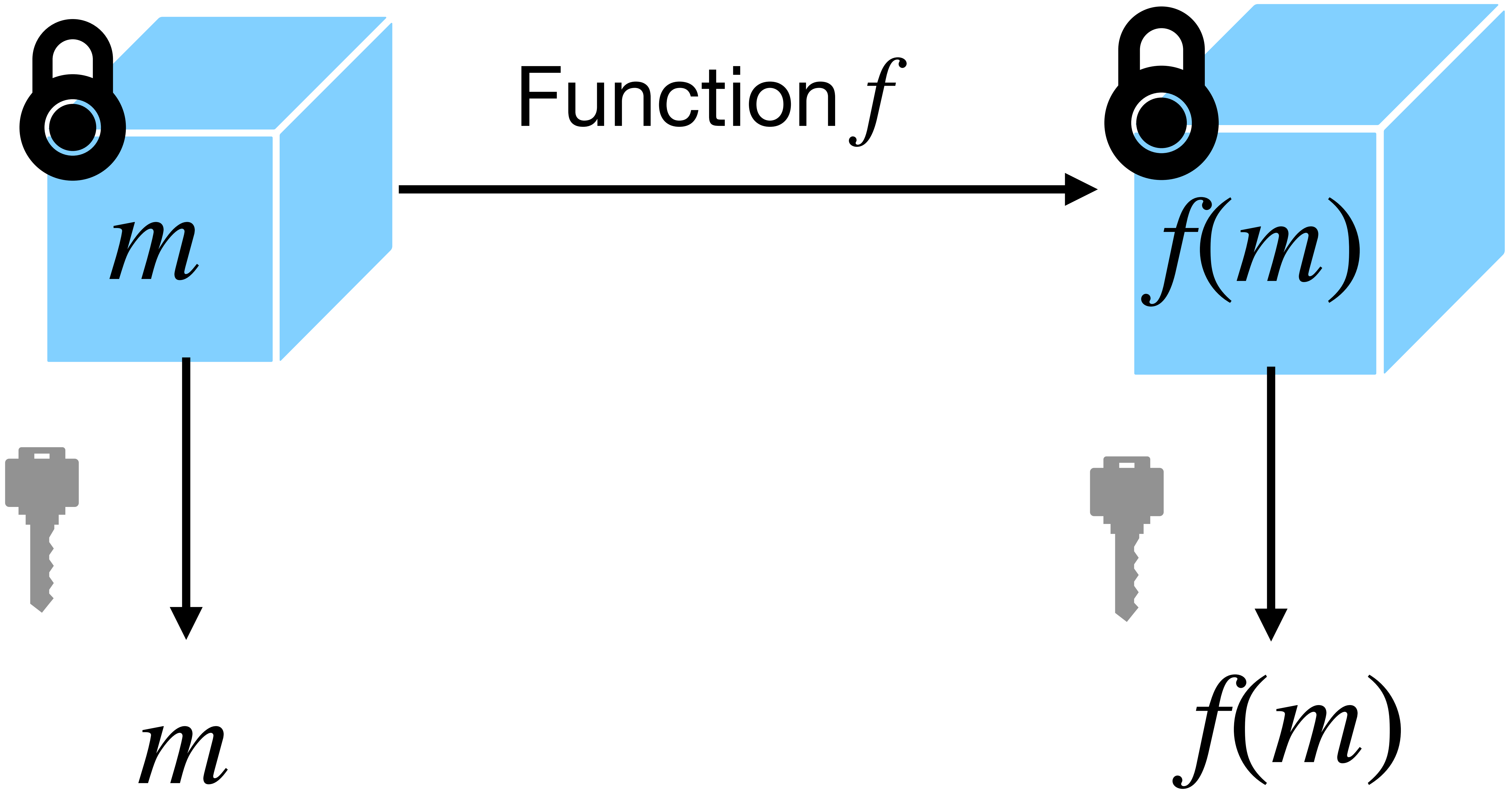

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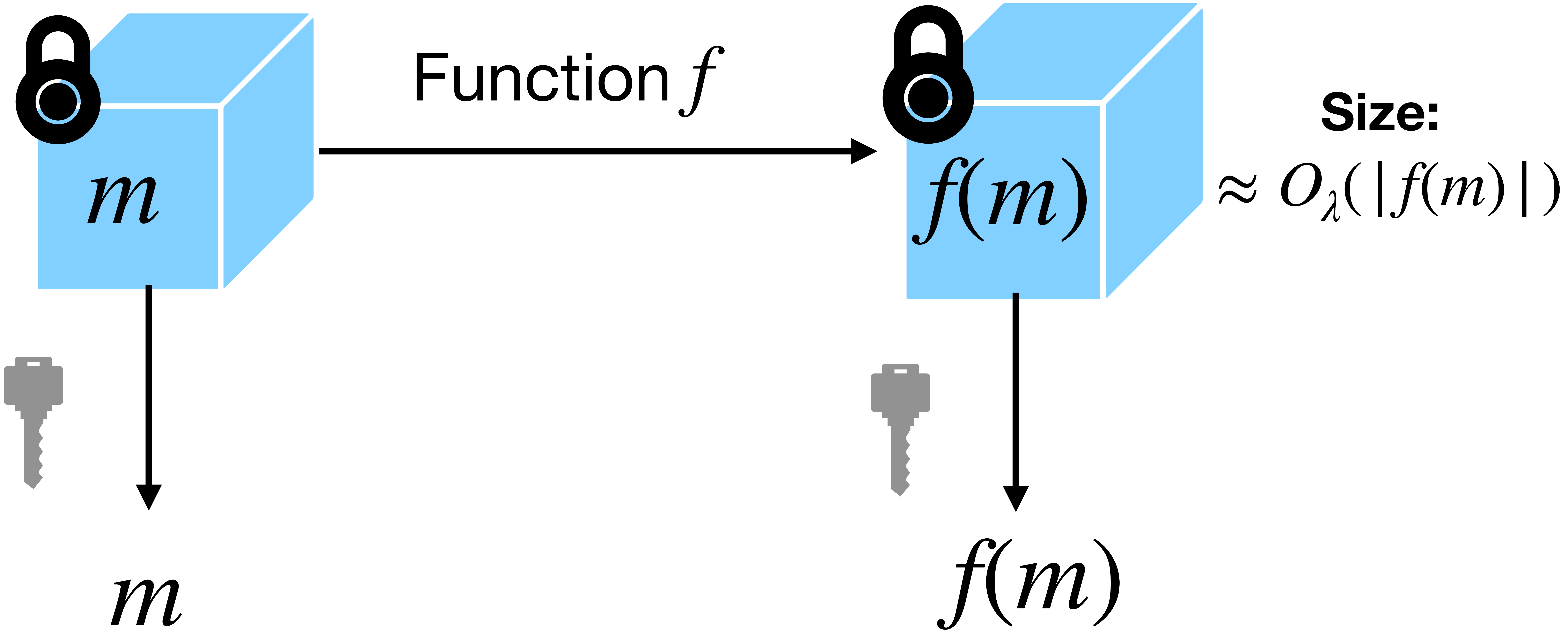

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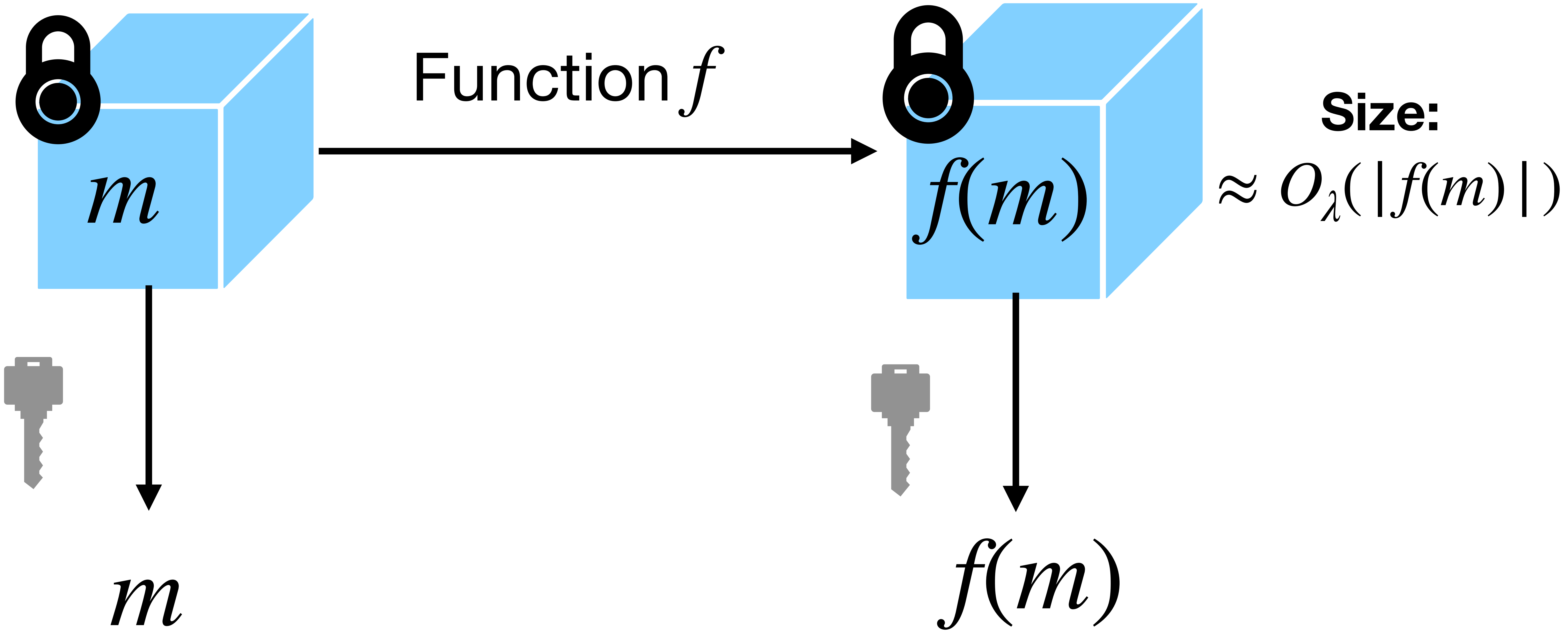


Theorem [G09, BV11]. Assuming polynomial hardness LWE, there exist (leveled) FHE.

KRR14 Construction

Based on [Biehl-Meyer-Wetzel ’98]

$\mathscr{P}$

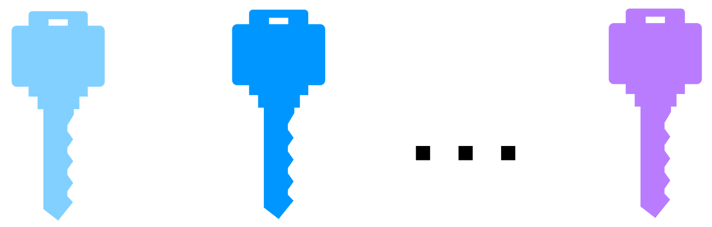
$\mathscr{V}$

KRR14 Construction

Based on [Biehl-Meyer-Wetzel '98]

$\mathscr{P}$

$\mathscr{V}$

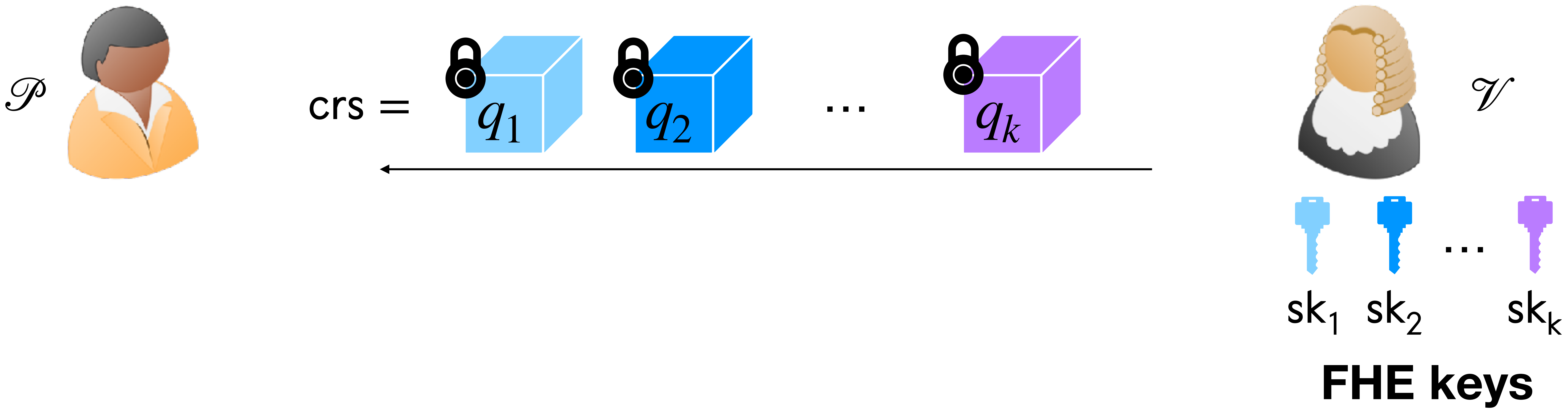


sk_1 sk_2 … sk_k

FHE keys

KRR14 Construction

Based on [Biehl-Meyer-Wetzel '98]



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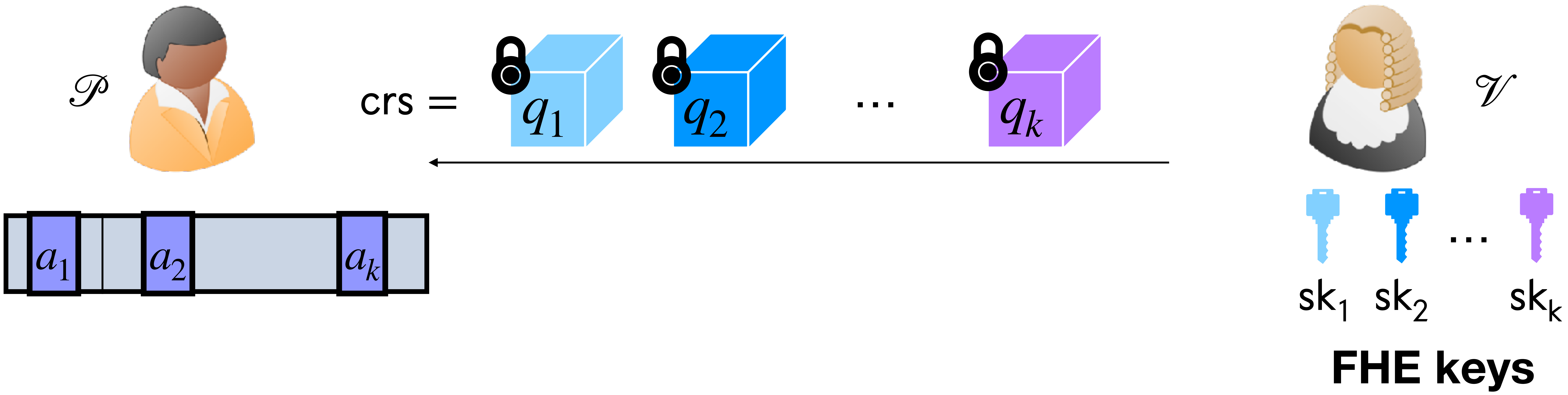

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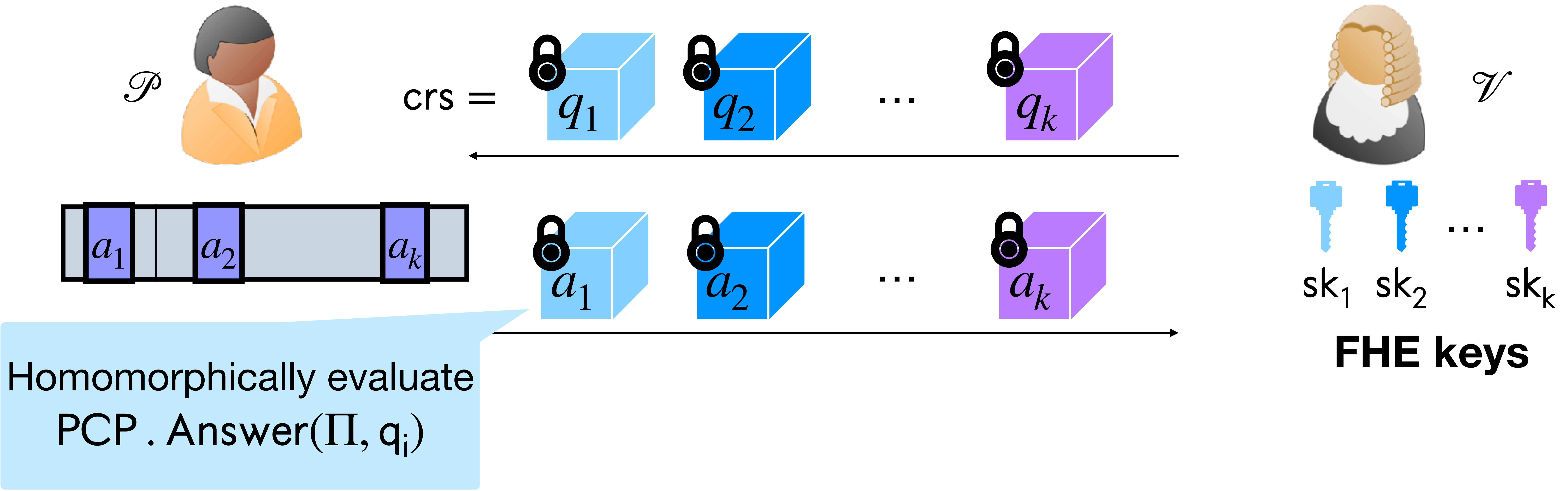

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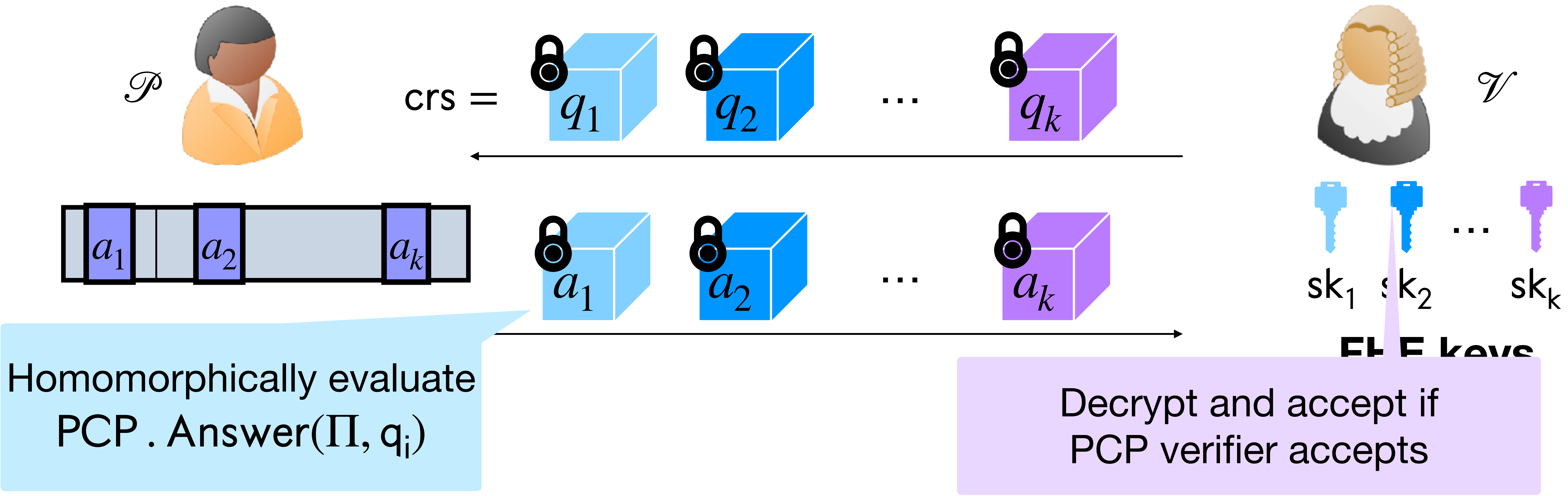

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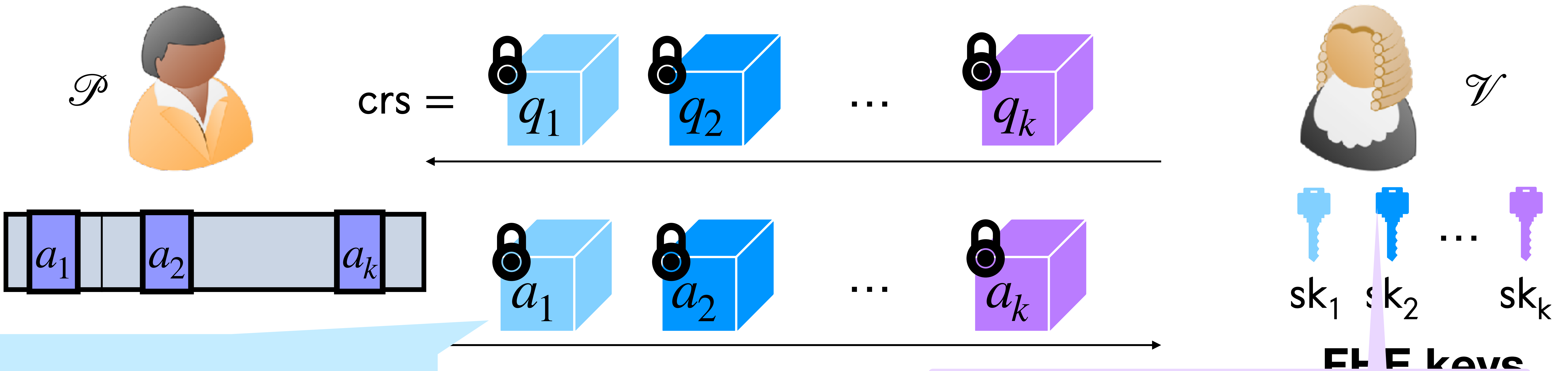


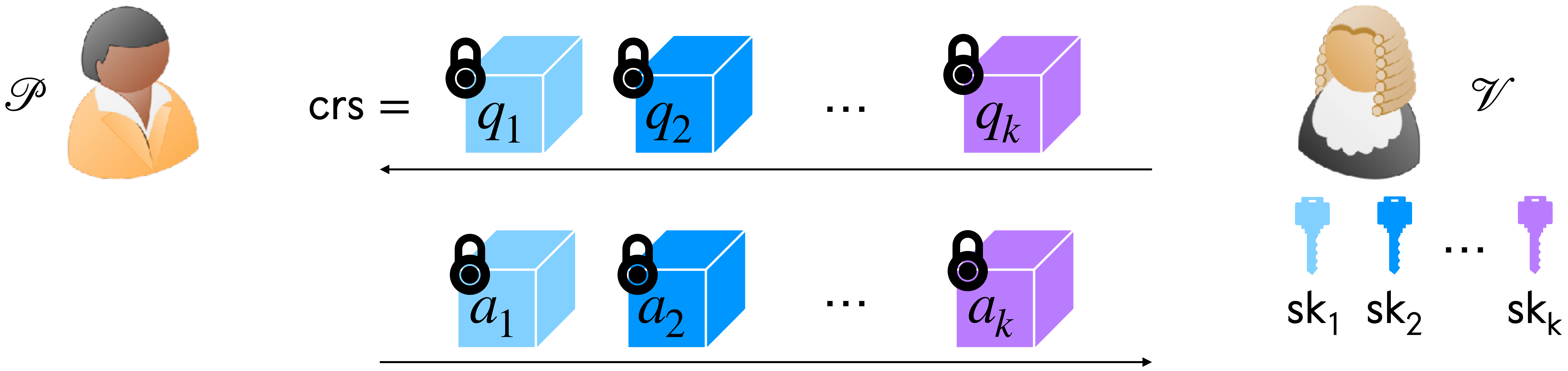
Homomorphically evaluate $\mathsf{PCP.Answer}(\Pi, q_i)$

Decrypt and accept if PCP verifier accepts

Intuition: How can $\mathscr{P}$ **cheat** if he doesn’t know what is being queried (FHE security)?

KRR14 Construction

Based on [Biehl-Meyer-Wetzel ’98]



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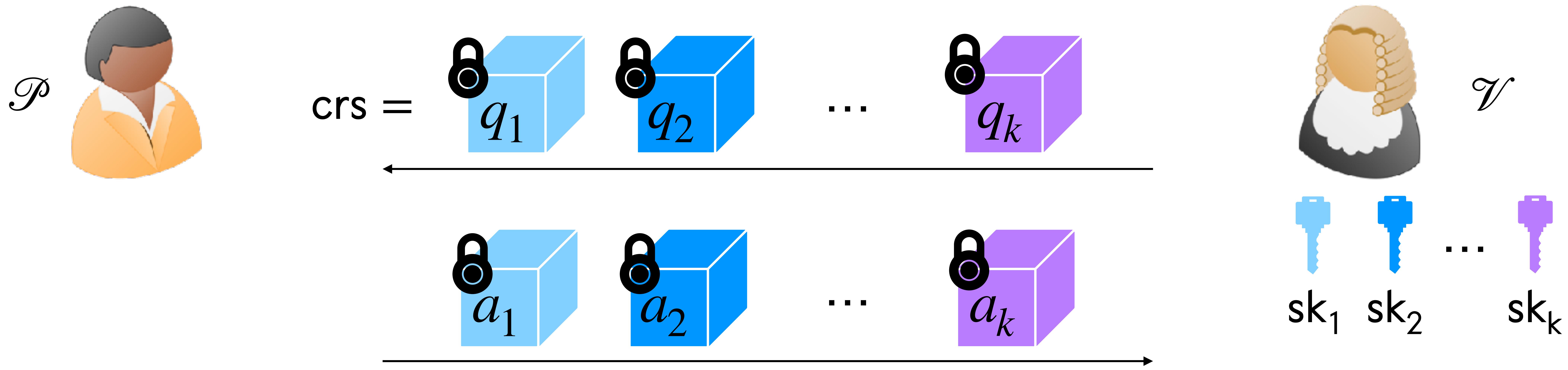

Issue 1: Need a **secret key** to verify (can be solved using [JKL**M**25])

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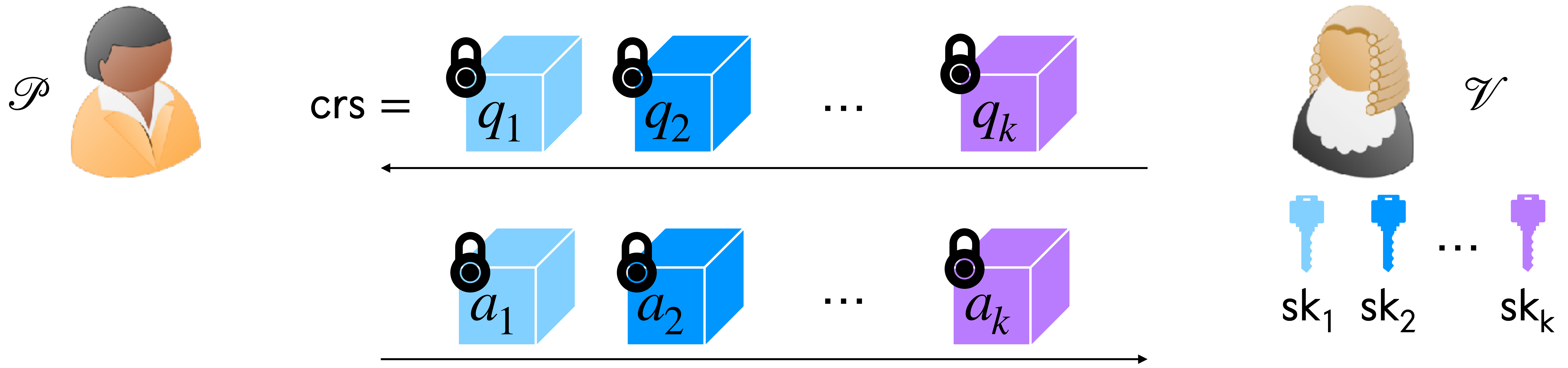

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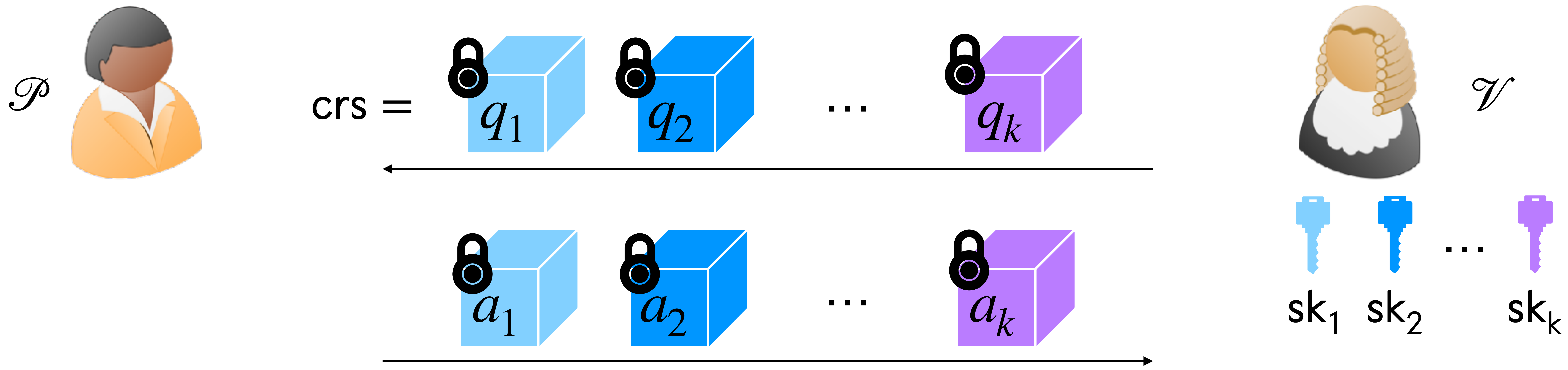

Issue 1: Need a **secret key** to verify (can be solved using [JKL**M**25])

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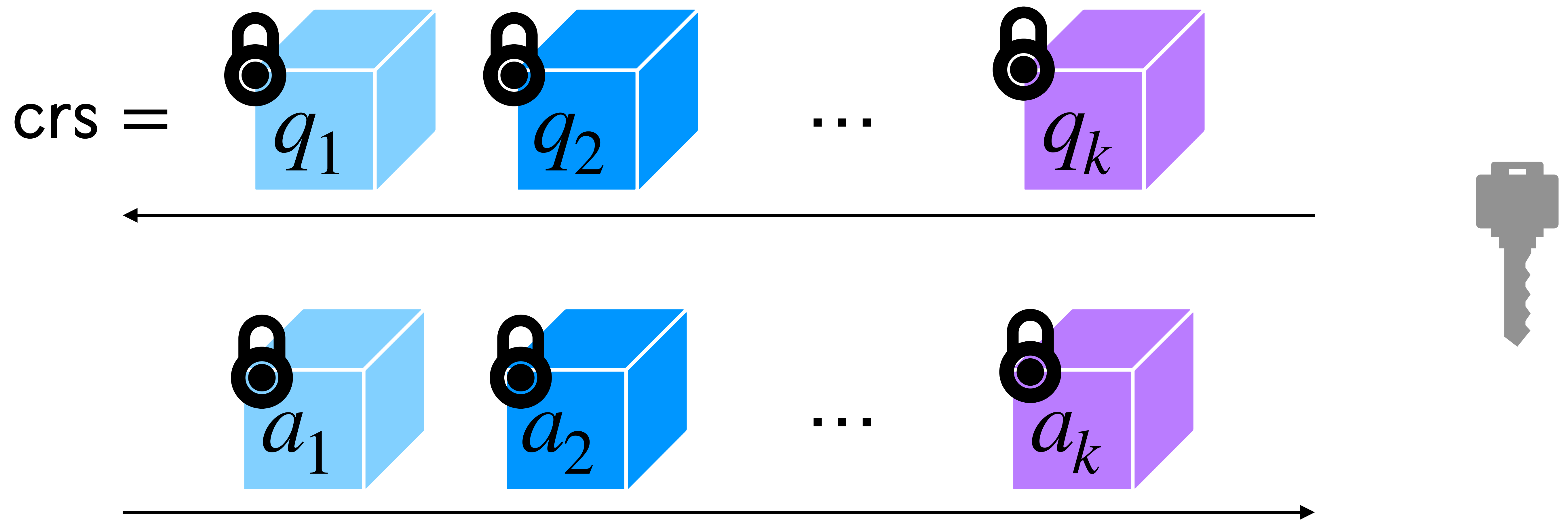
- Still runs into the problem that the verifier is not committed to **one** Π!
  (Bonus slide demonstrating this)

KRR14 Construction

Based on [Biehl-Meyer-Wetzel ’98]



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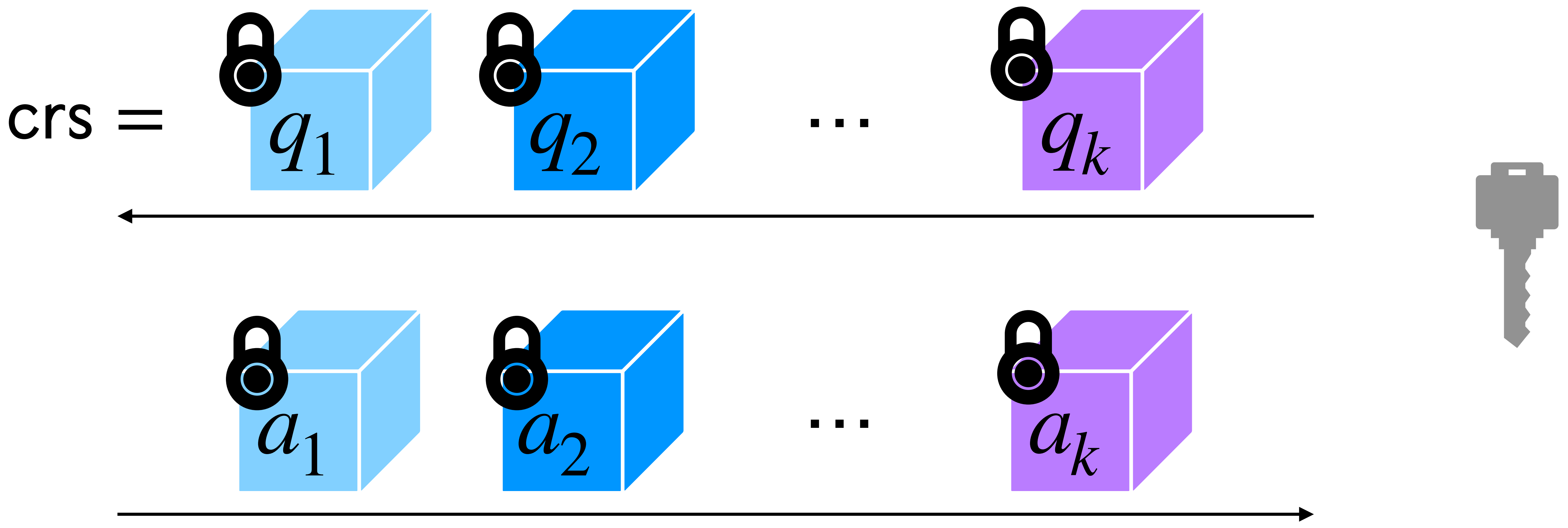

Independent keys: Adversary doesn’t “**see all**” $\{q_i\}_i$ anymore.

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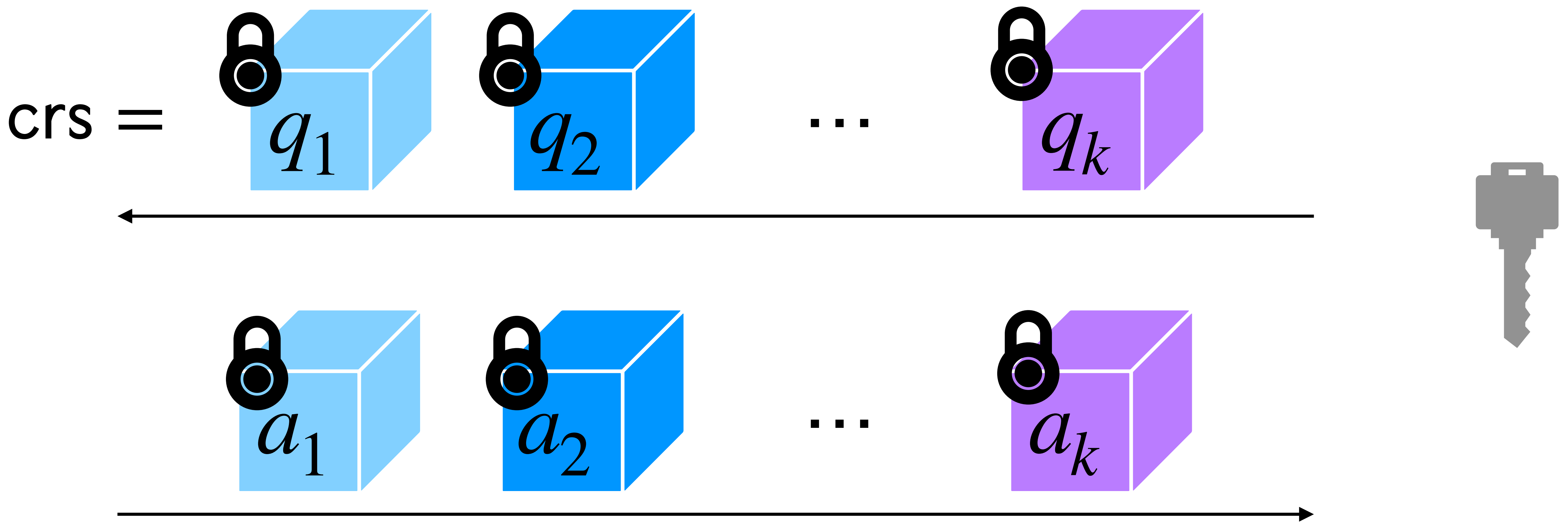

Independent keys: Adversary doesn't "**see all**" $\{q_i\}_i$ anymore.

i.e. "Information" should **not** be transmitted between answers.

KRR14 Guarantee

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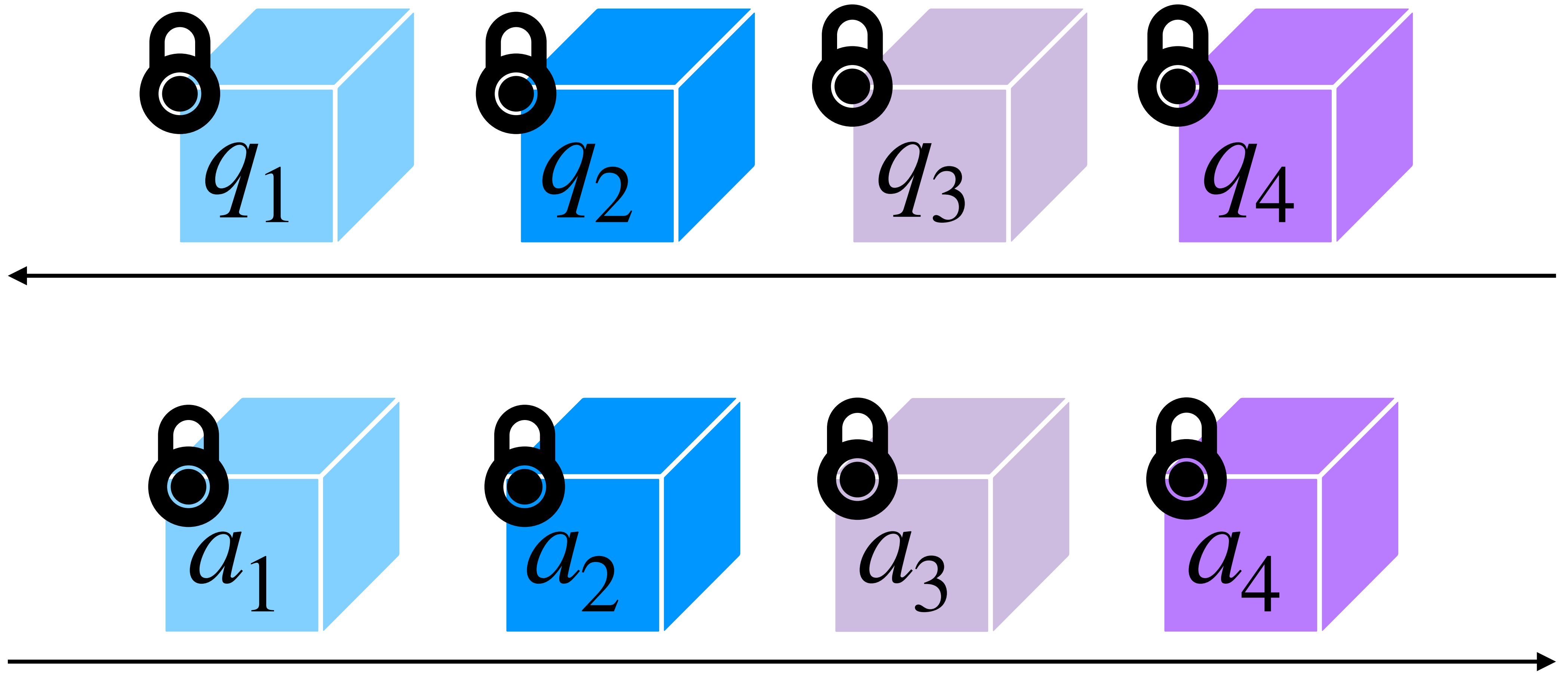

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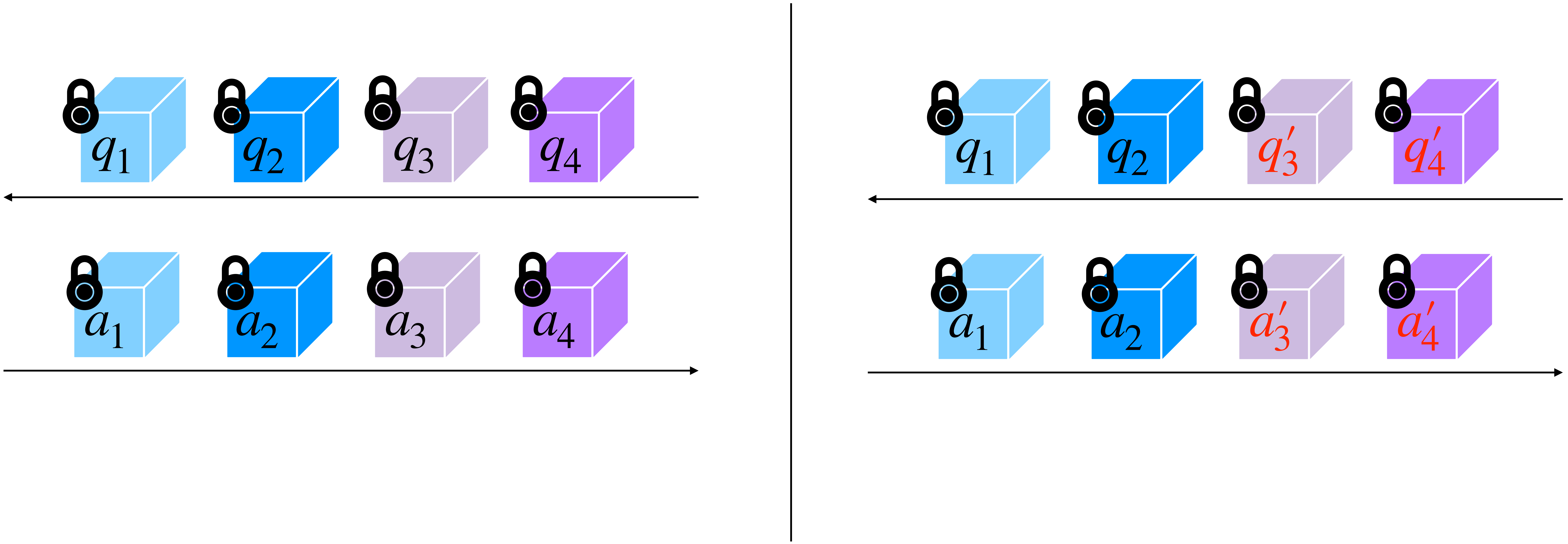

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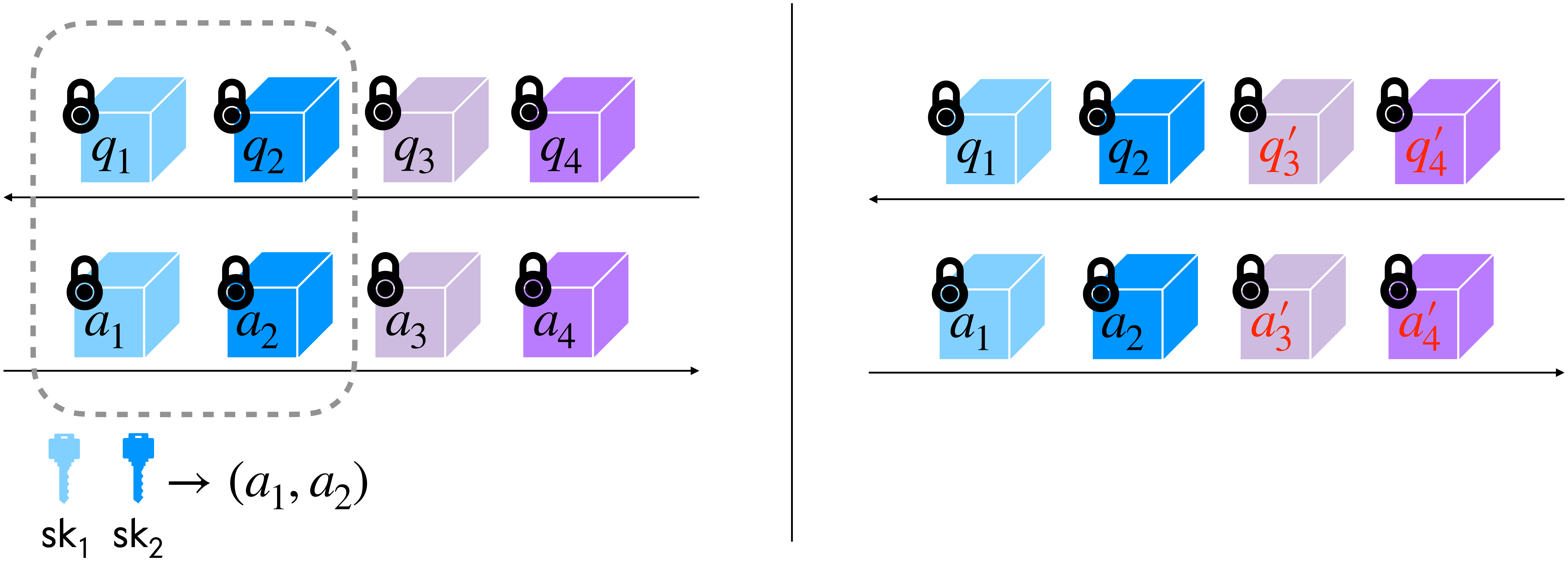

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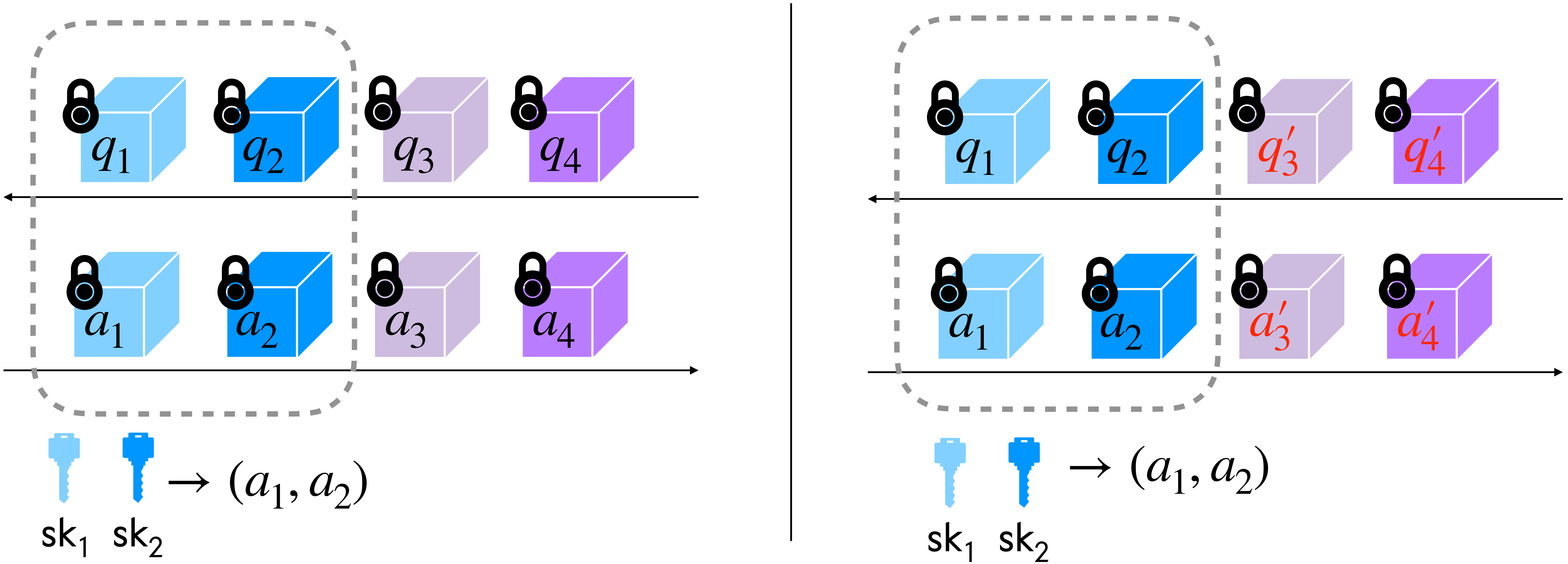

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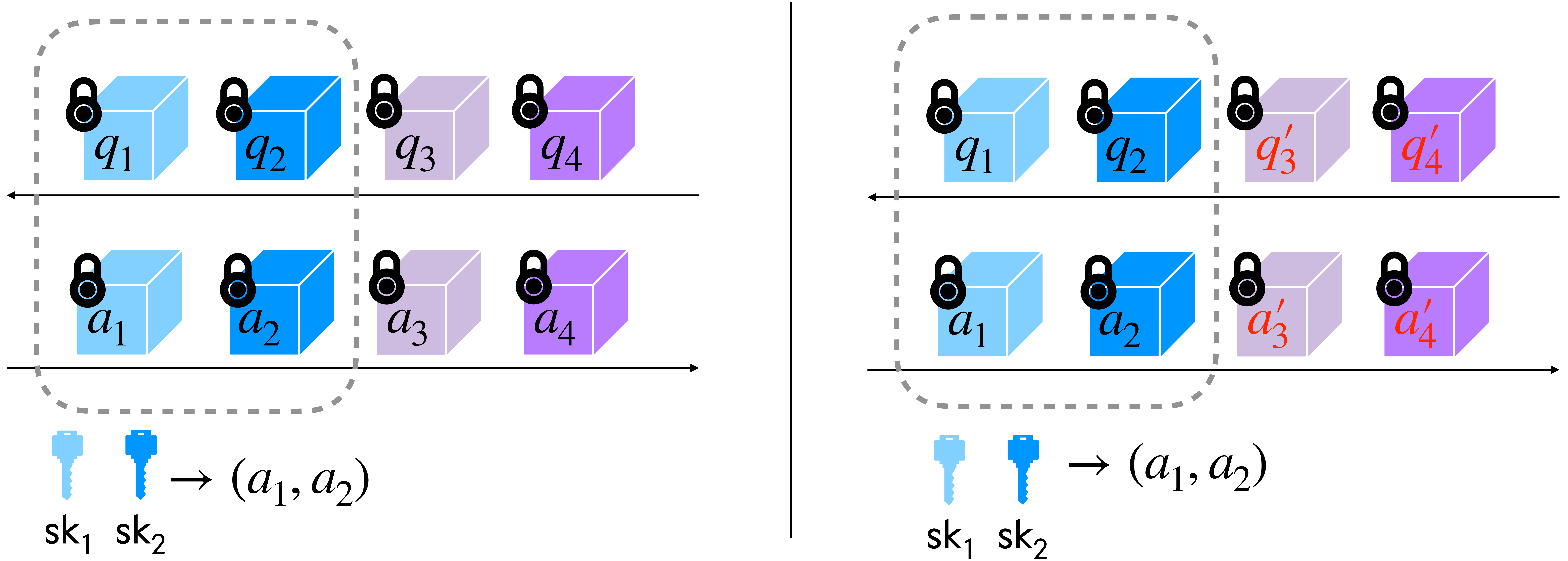

Semantic security of $(\mathsf{sk}_3, \mathsf{sk}_4)$:

PCPs answers (a_1, a_2) should be **indistinguishable** in both experiments.

Enter: Non-Signaling PCPs

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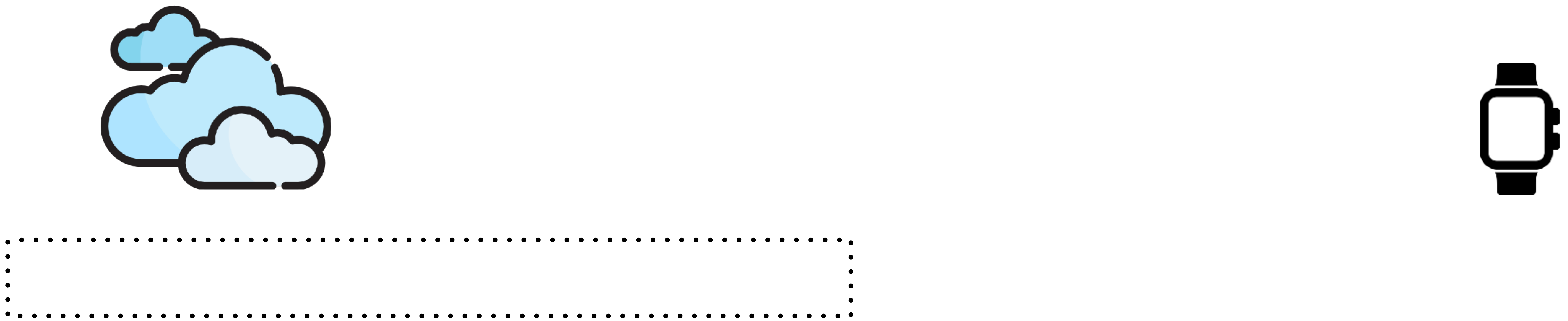

Enter: Non-Signaling PCPs

Family of distributions:

$$\mathscr{D} = \{D_Q\}_Q$$

Enter: Non-Signaling PCPs

Family of distributions:

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Q

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$A_Q \leftarrow D_Q$

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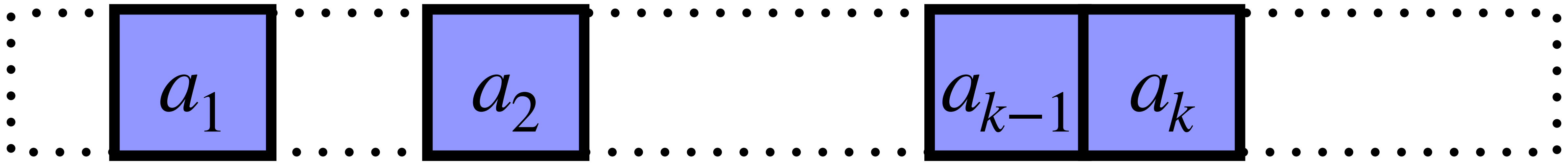

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Enter: Non-Signaling PCPs

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Q

A_Q

a_1 a_2 a_{k-1} a_k

$A_Q \leftarrow D_Q$

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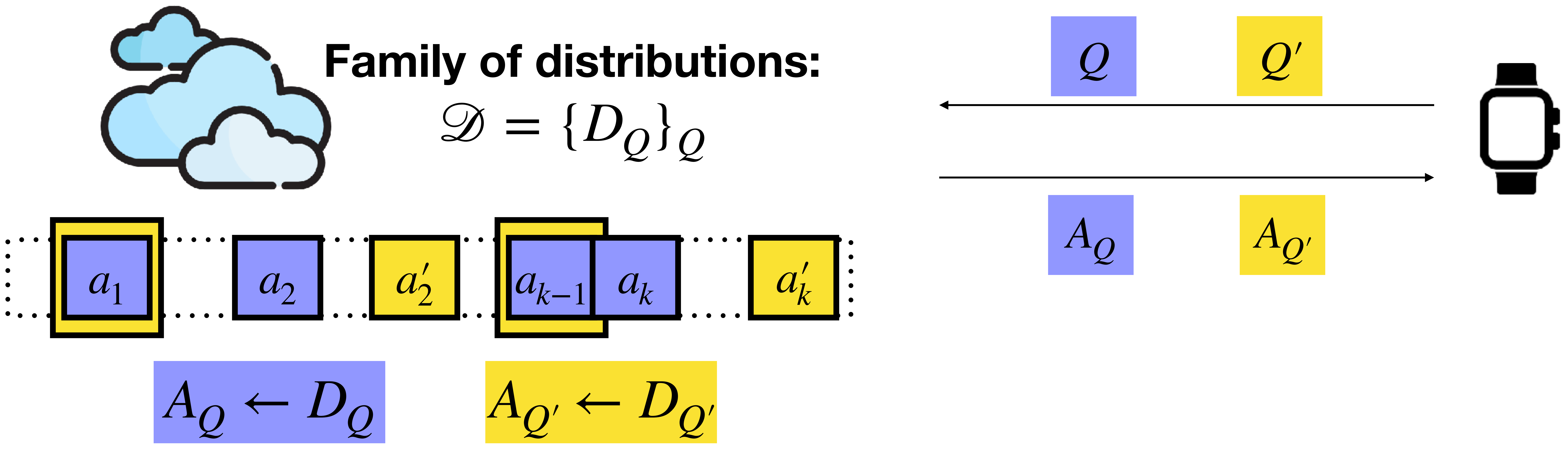

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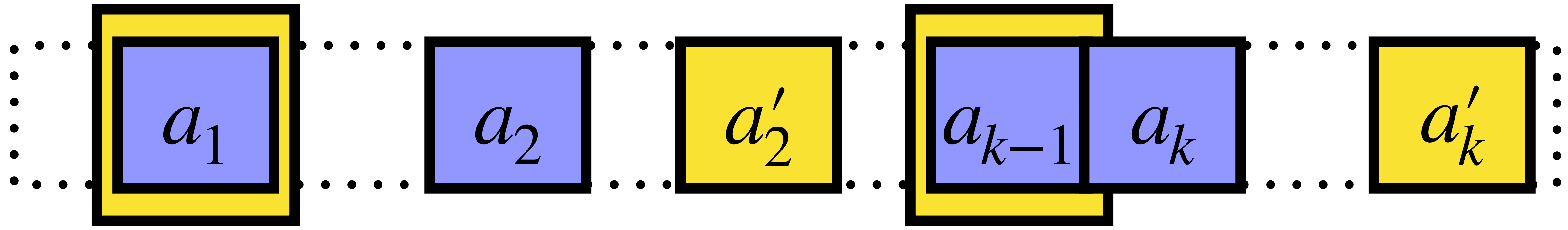
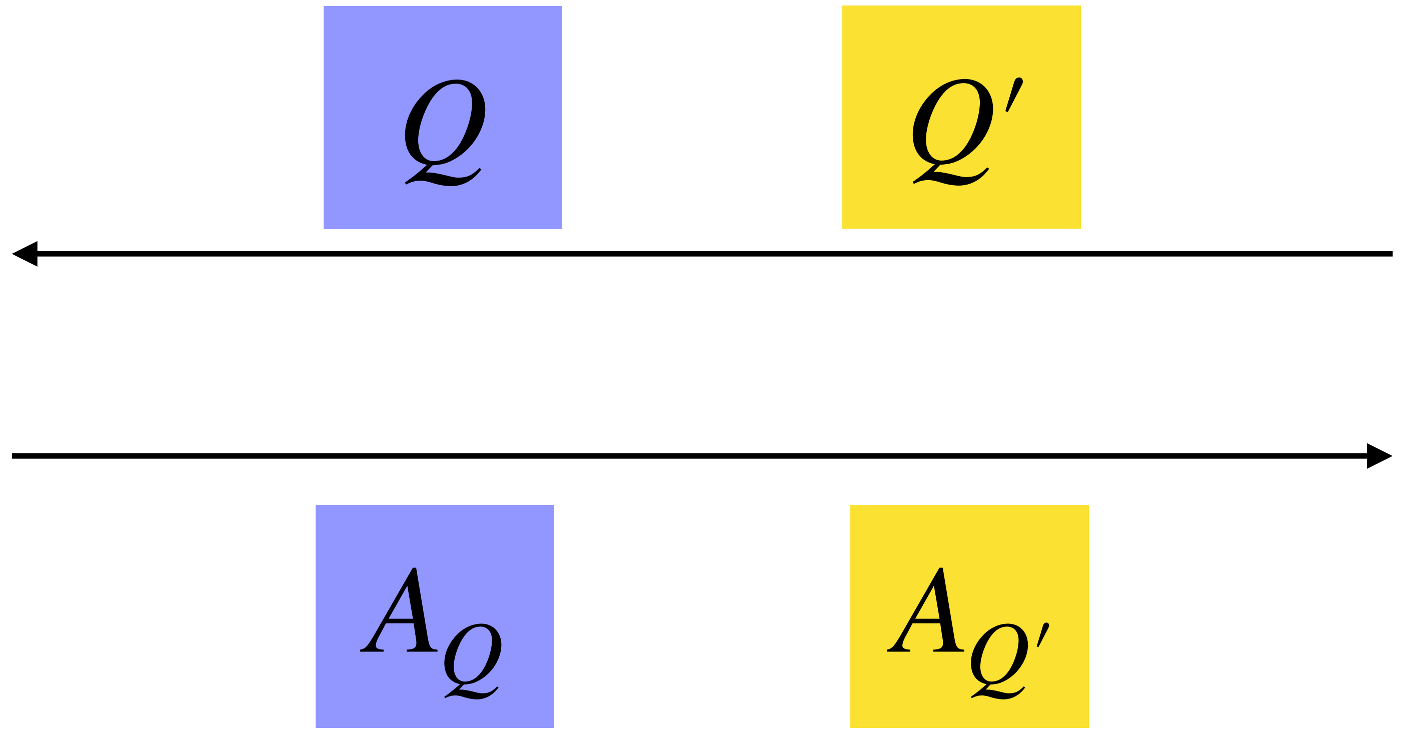


$A_Q \leftarrow D_Q$

$A_{Q'} \leftarrow D_{Q'}$



Non-signaling: Let $S = Q \cap Q'$.

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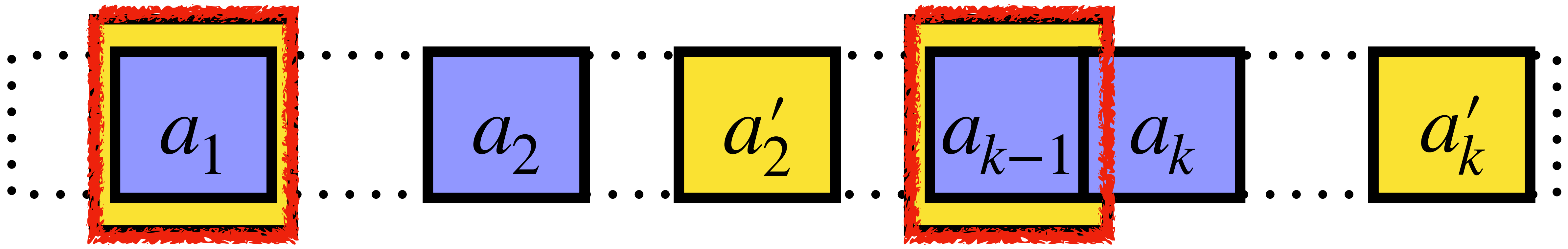

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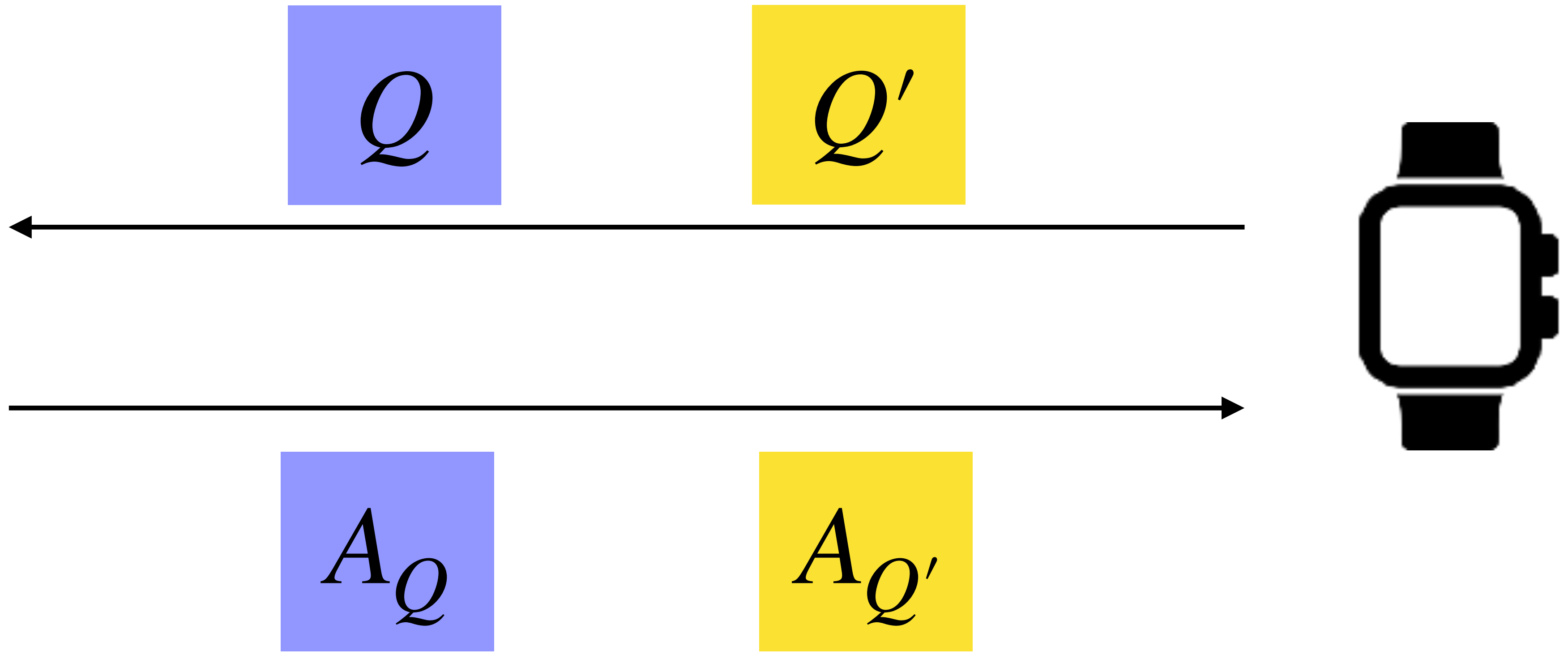

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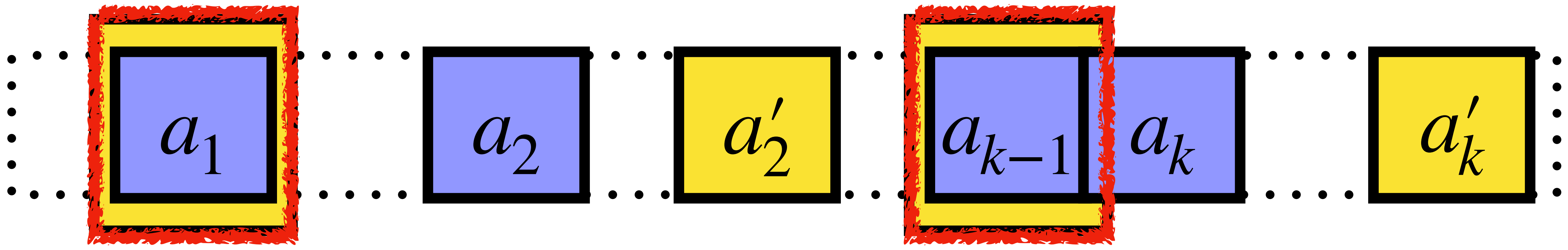

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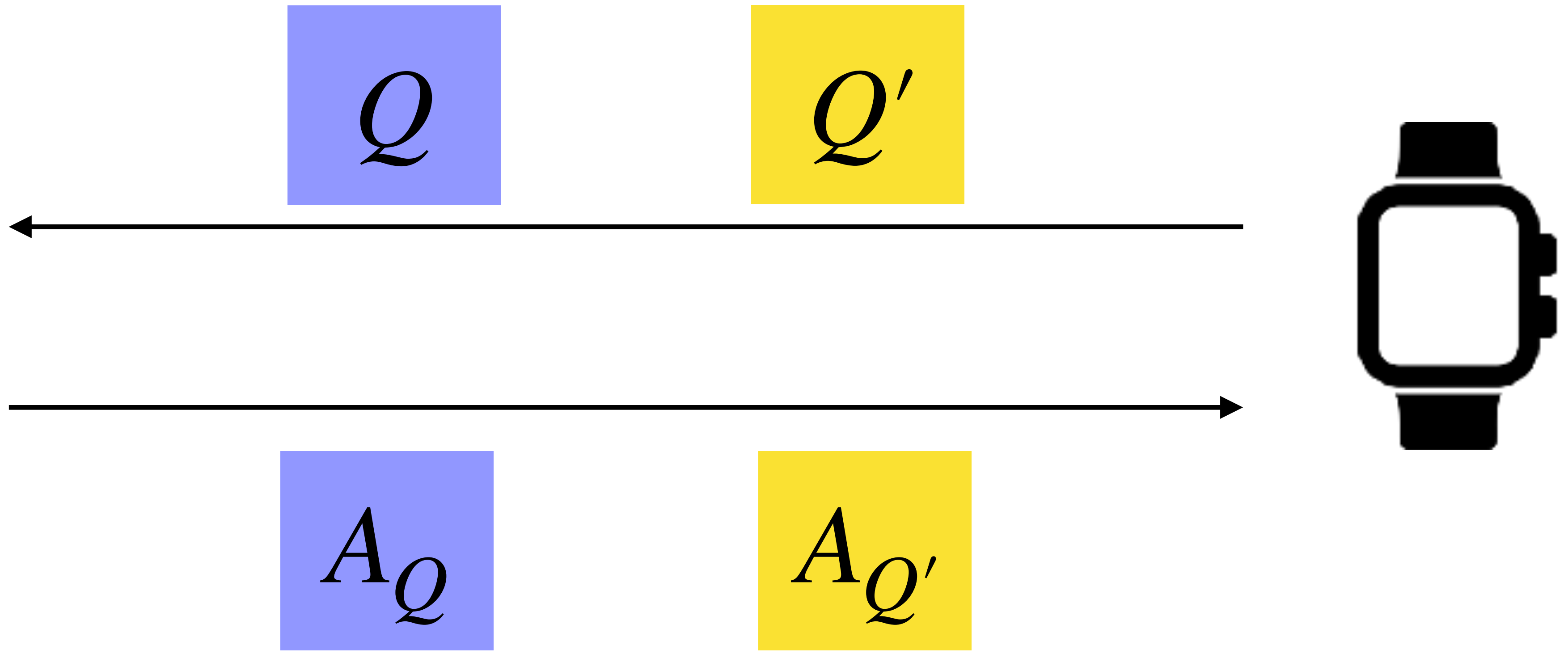

NS Soundness: If $x \notin \mathscr{L}$
$$\Pr_{Q,A_Q}[V(x, Q, A_Q) = 1] \leq \frac{1}{\text{poly(n)}}$$

Enter: Non-Signaling PCPs

[Alternate view]

Non-signaling:

$S = Q \cap Q'$.

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Arbitrary correlation that obeys "locality"

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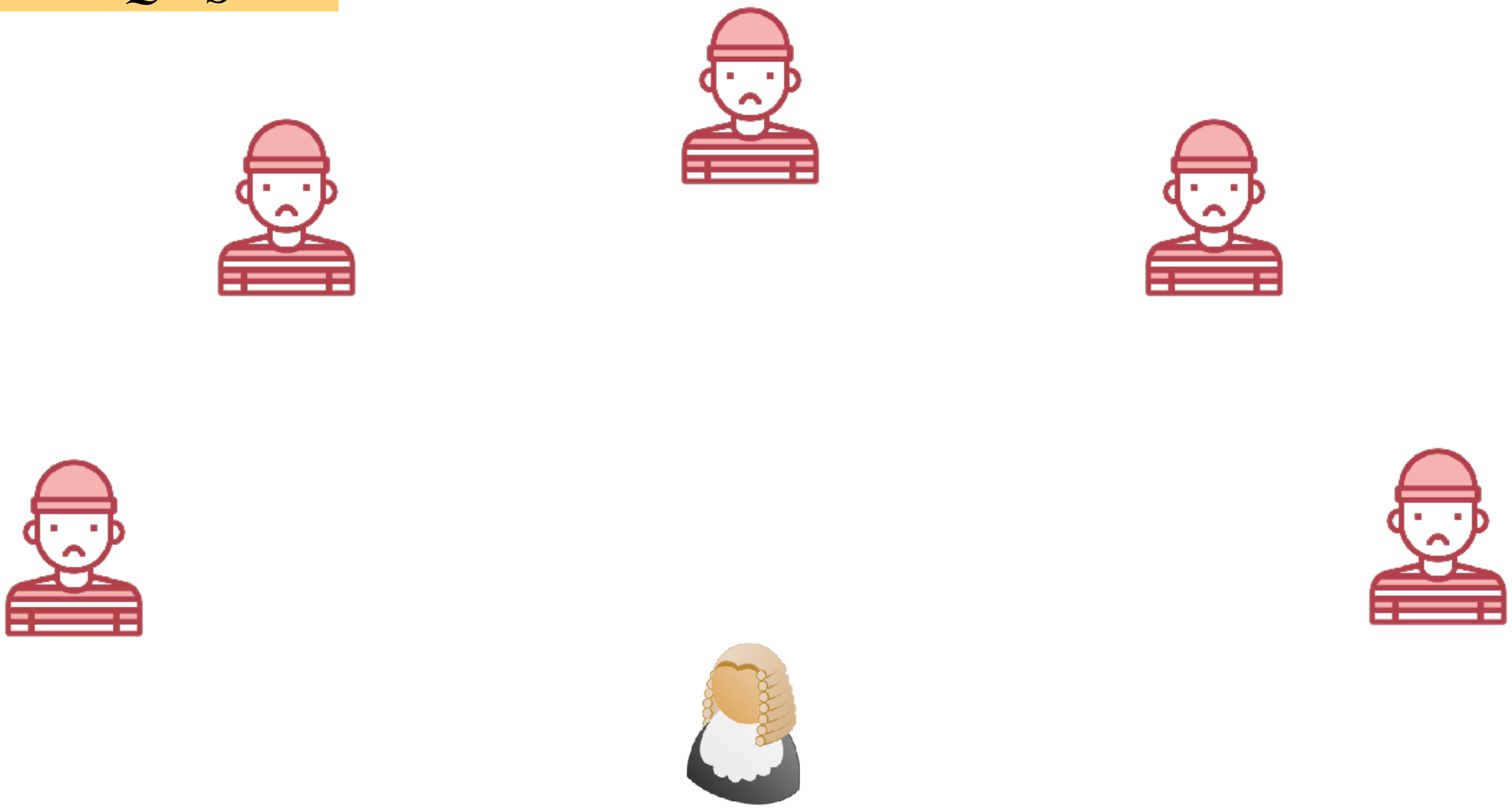

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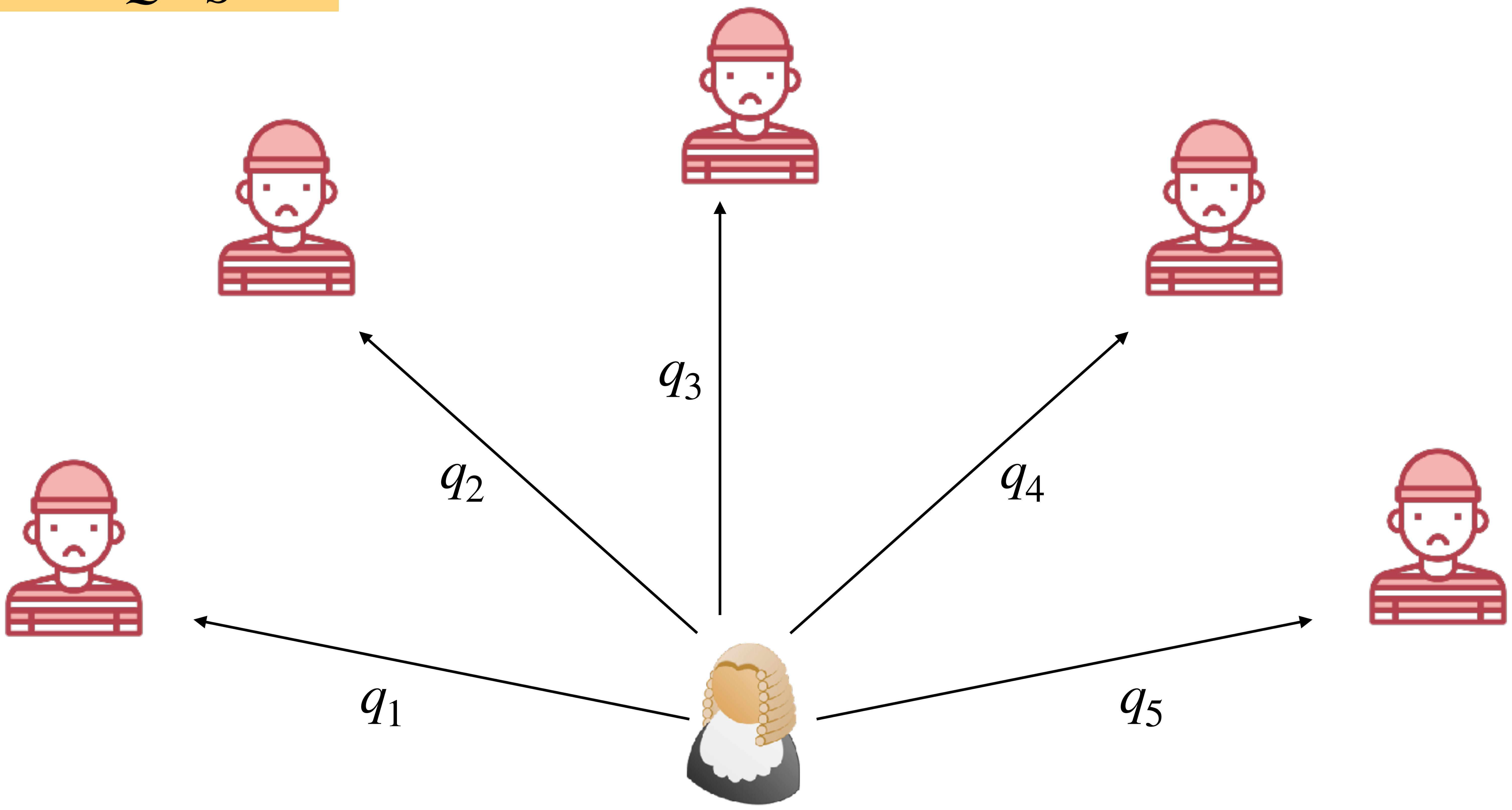

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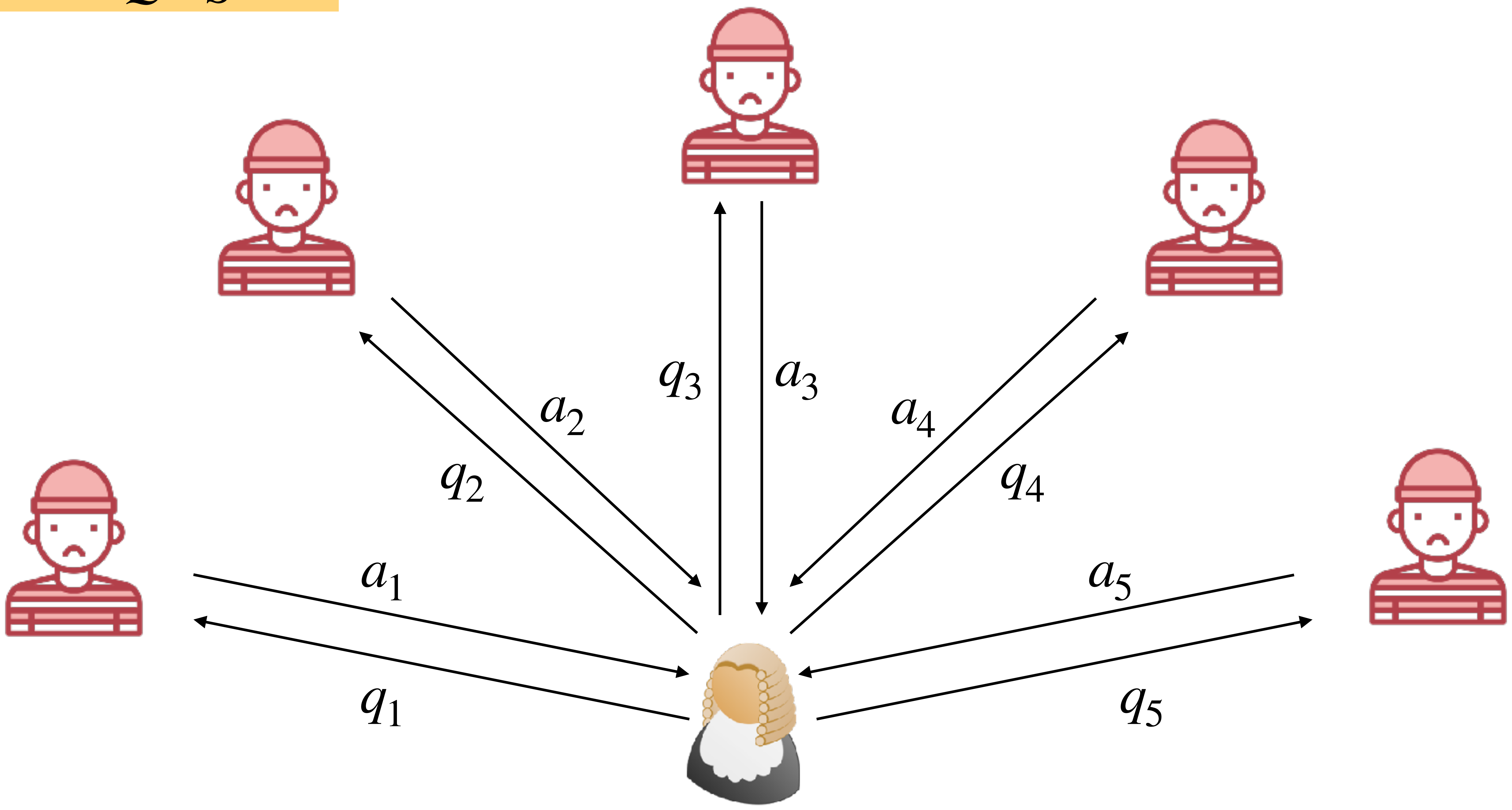

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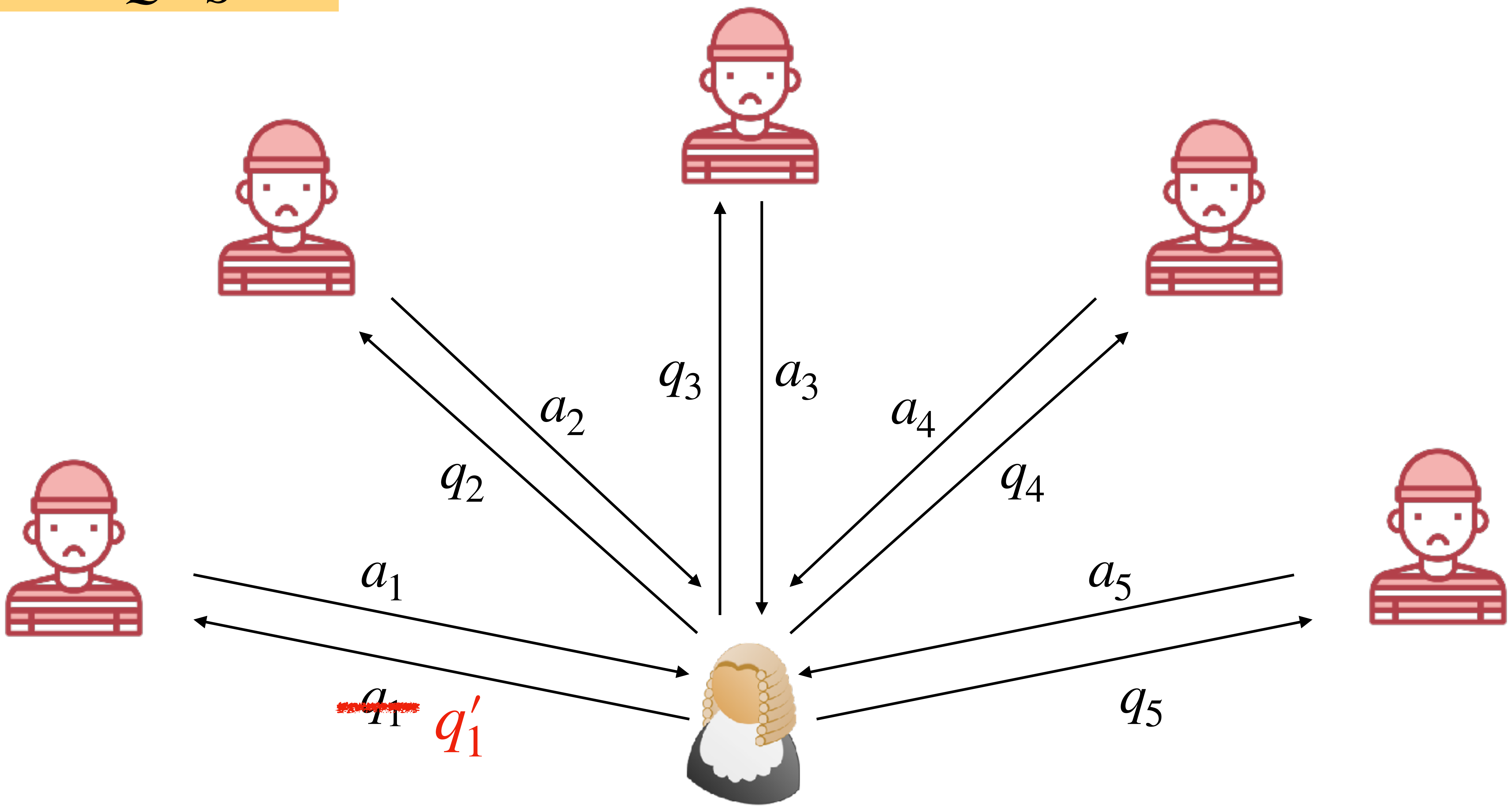

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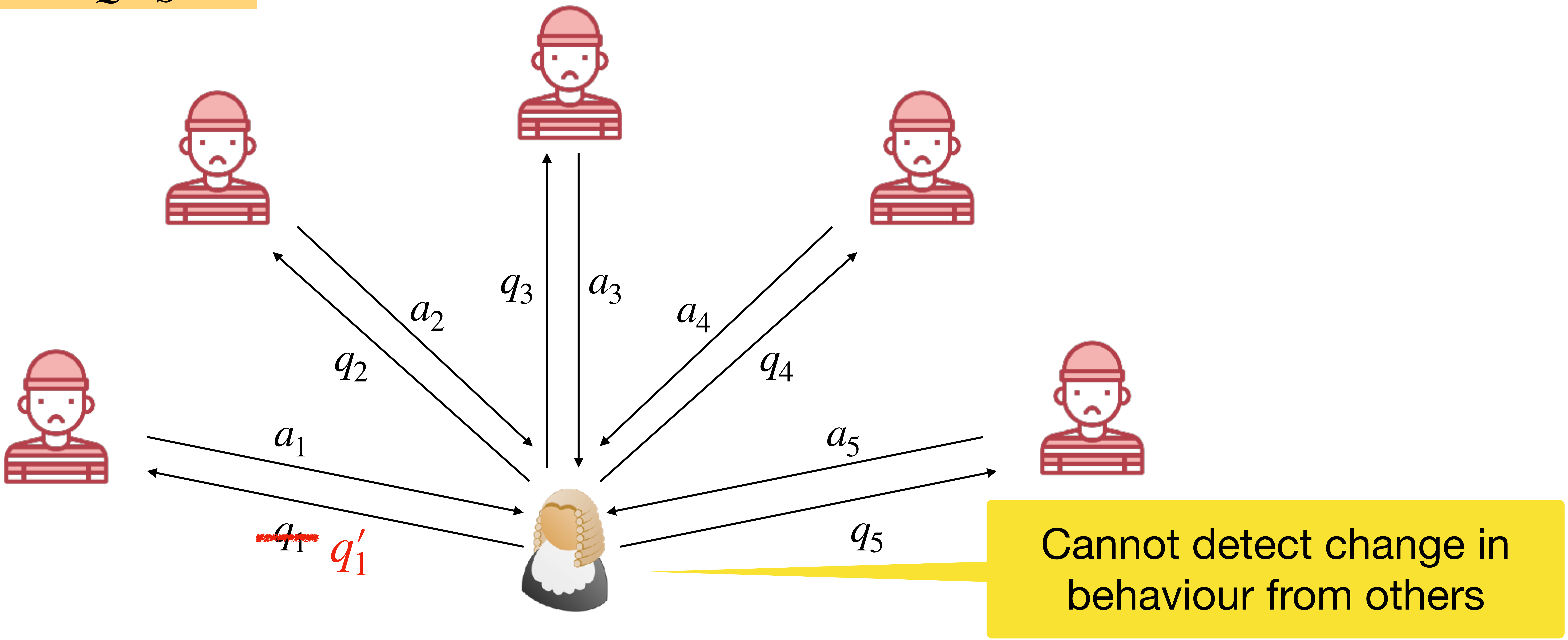

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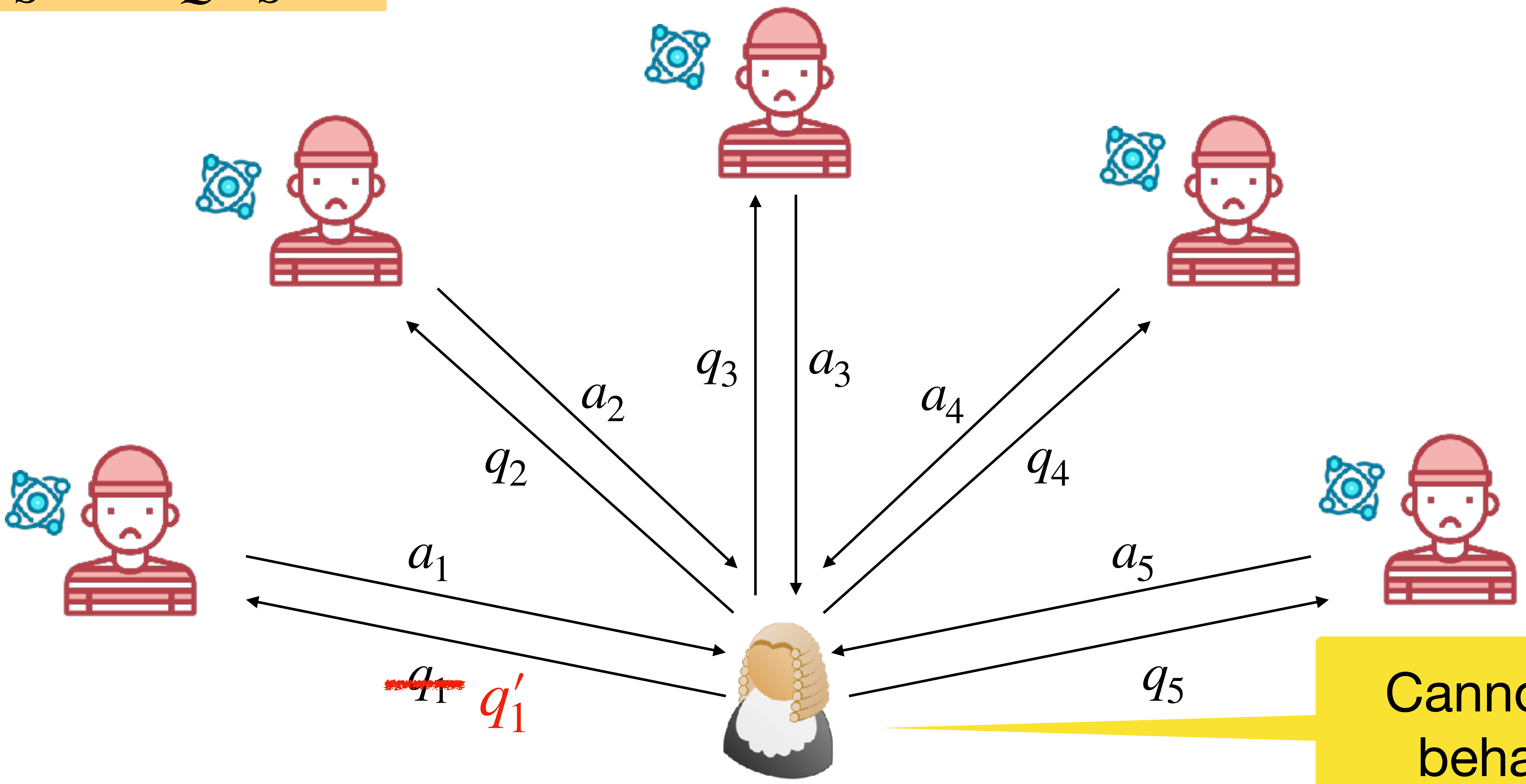

Cannot detect change in behaviour from others

Enter: Non-Signaling PCPs

[Alternate view]

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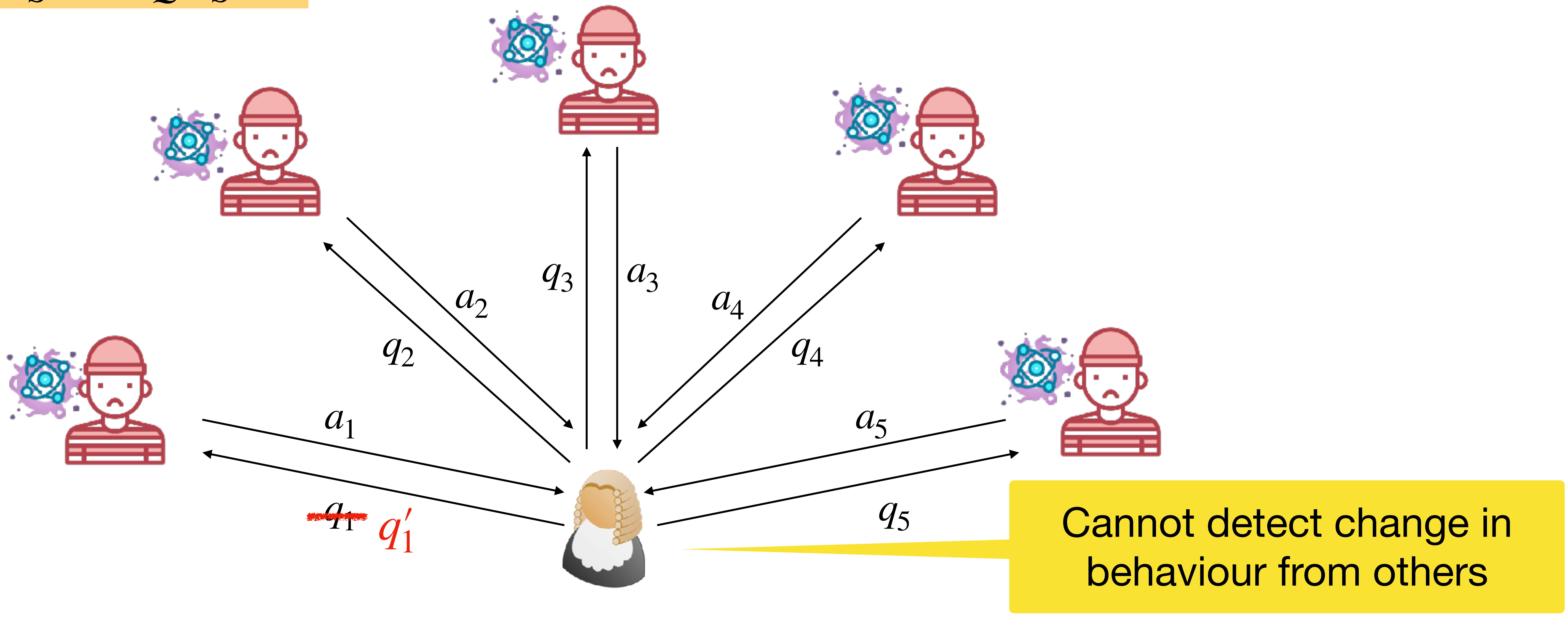

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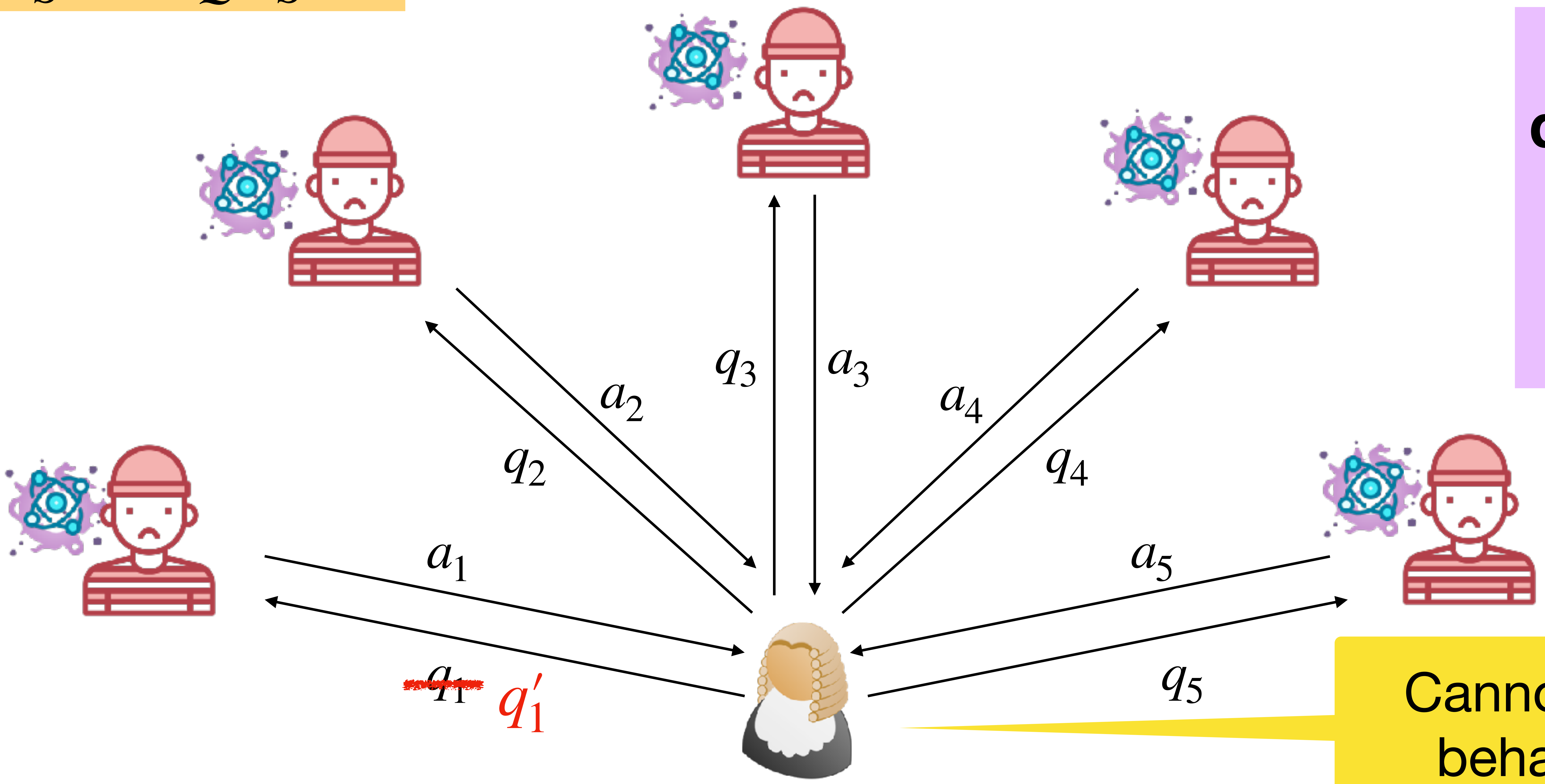
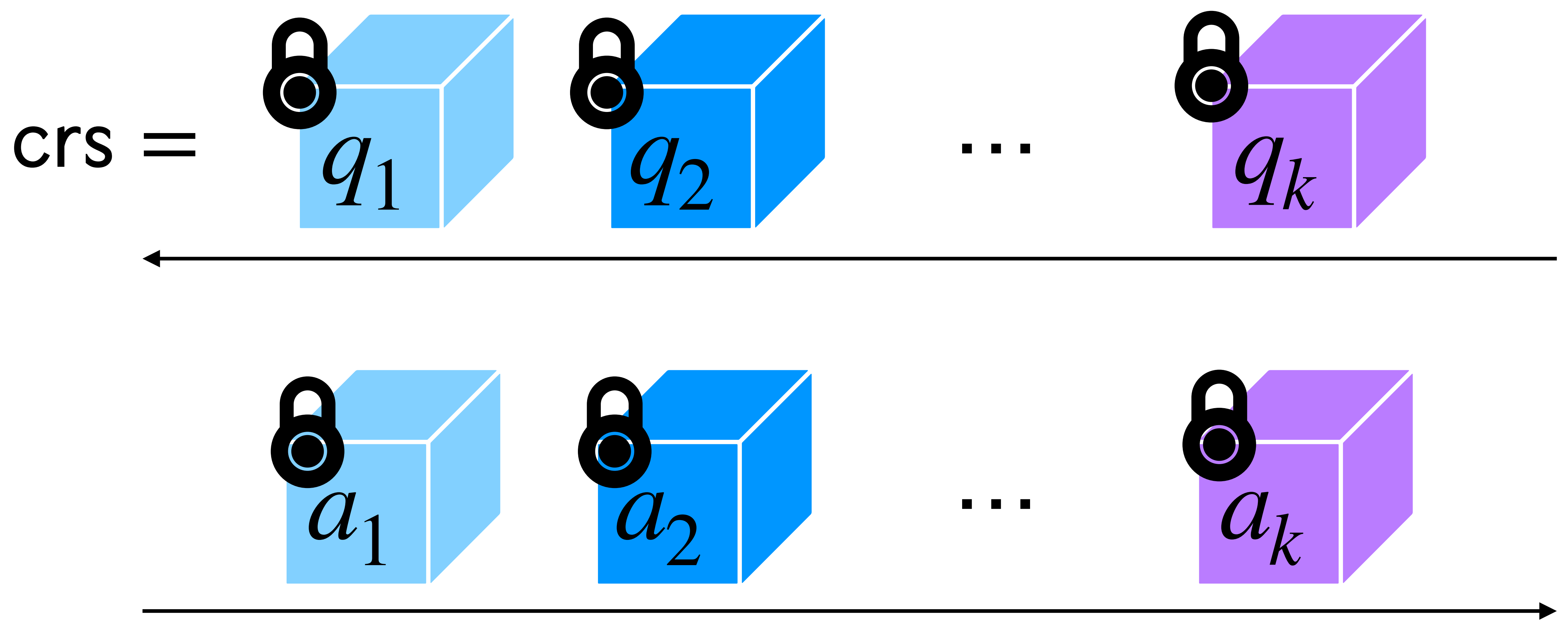


Generalization of quantum strategies.
E.g. CHSH game.
Quantum: ~0.85
NS: 1

Cannot detect change in behaviour from others

KRR Construction



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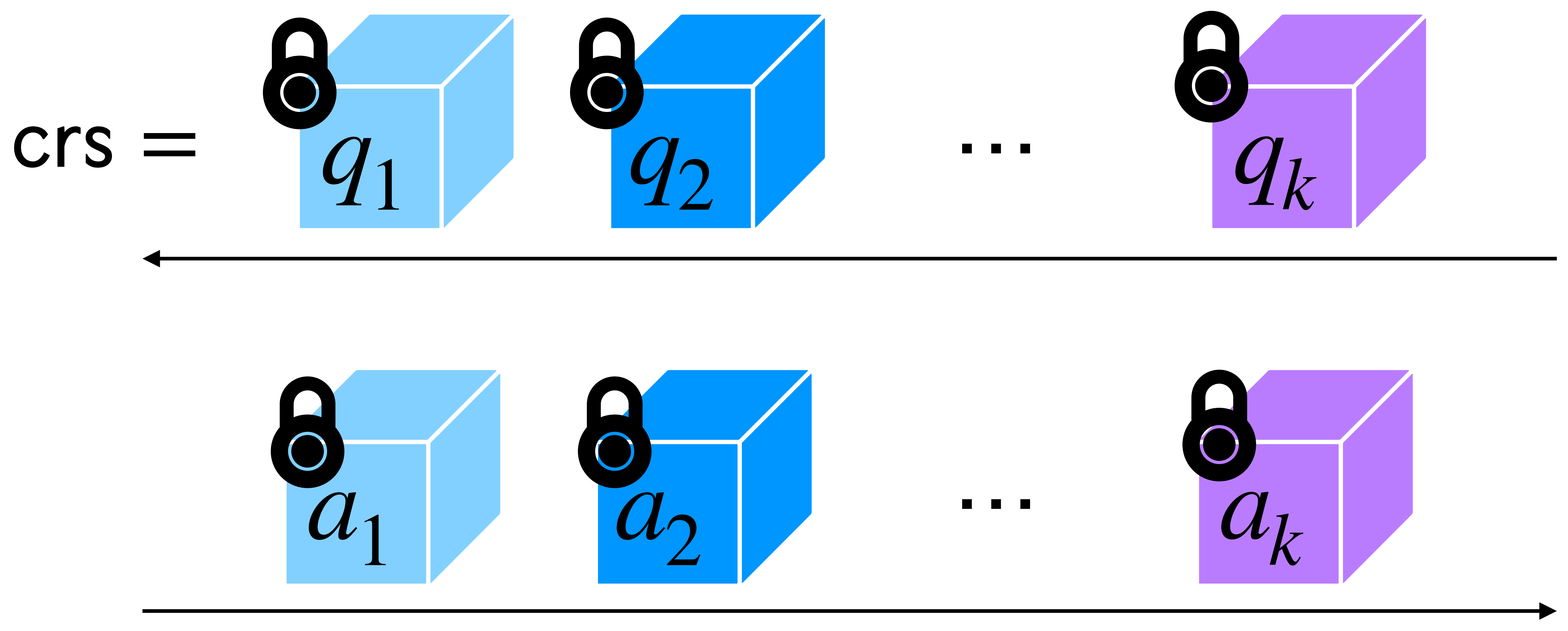
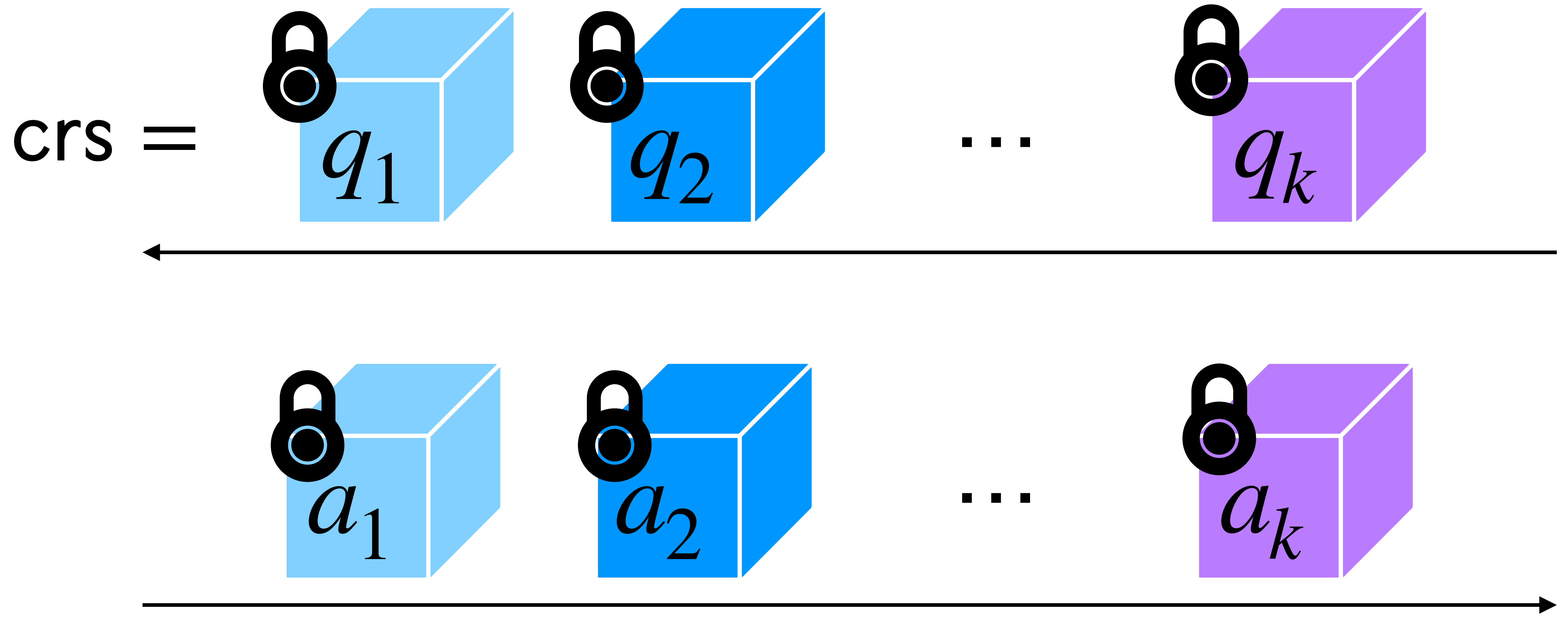


Theorem statement:
FHE Security + NS-PCP = KRR Secure

Pf. Cheating $\mathscr{A}$ gives an NS PCP strategy ∎

KRR Construction



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We have nsPCPs for:

Deterministic Computations
[KRR14, BHK17, CJJ21ab, KVZ21, …]

NTISP [BKK+17, KVZ21]

Batch-NP [CJJ21ab, WW22, …]

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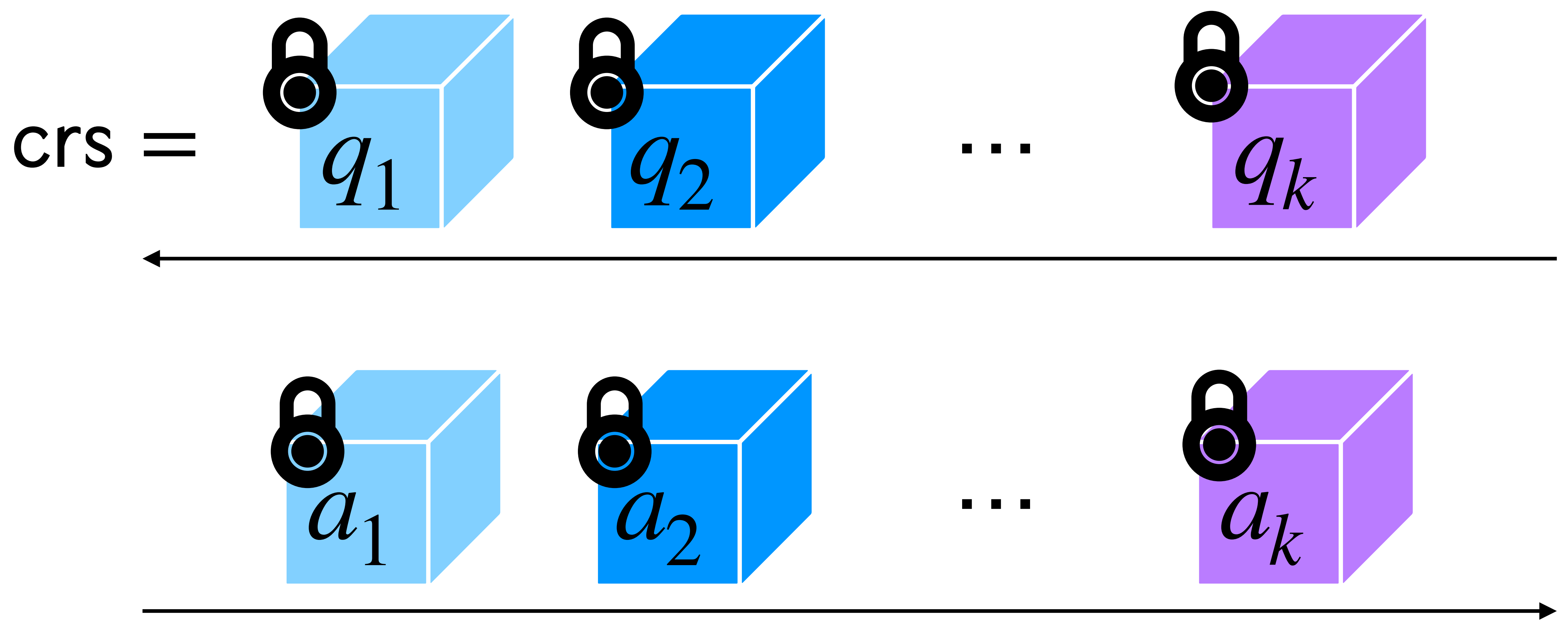

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What about NP?

Any questions so far?

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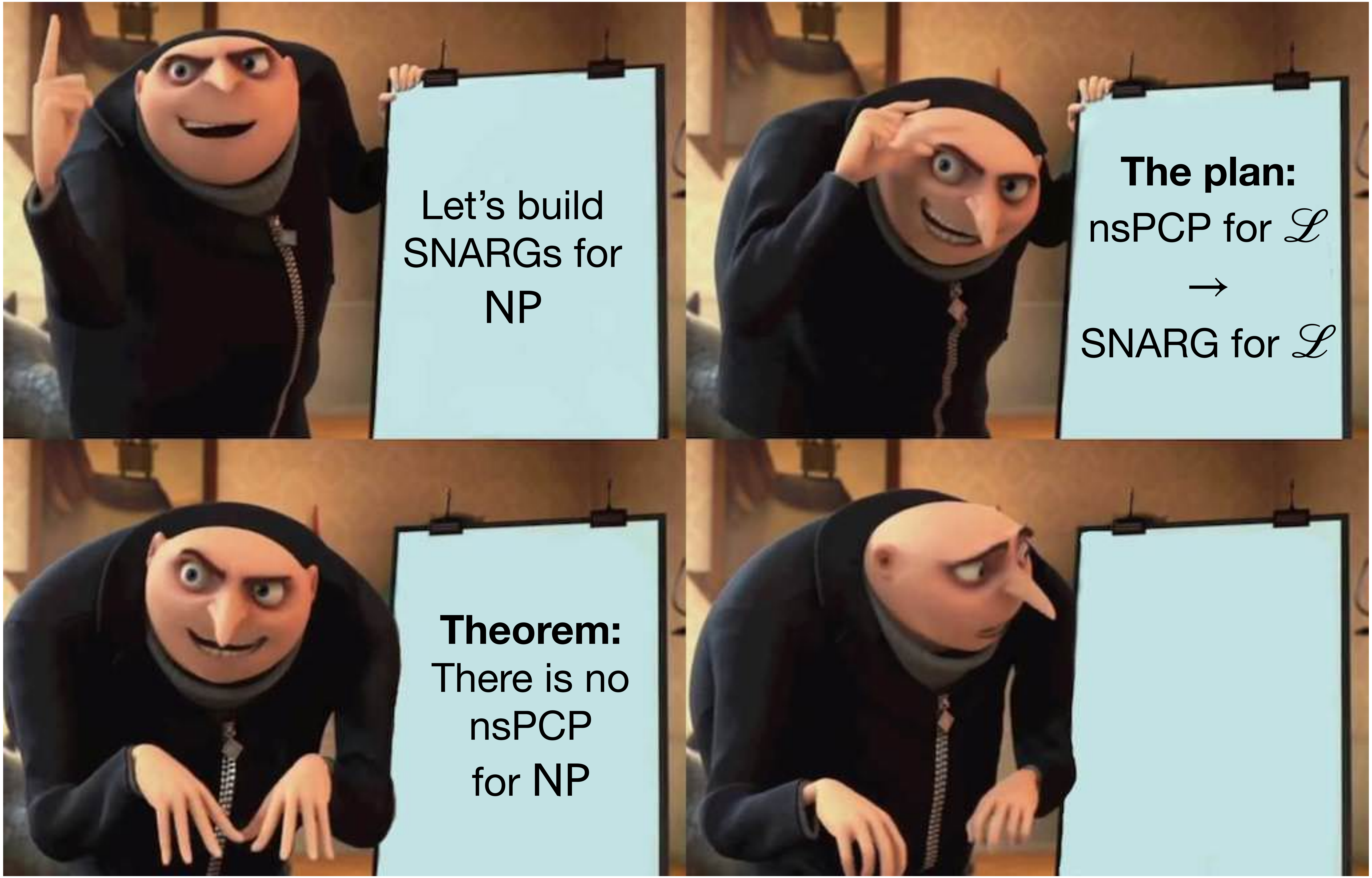
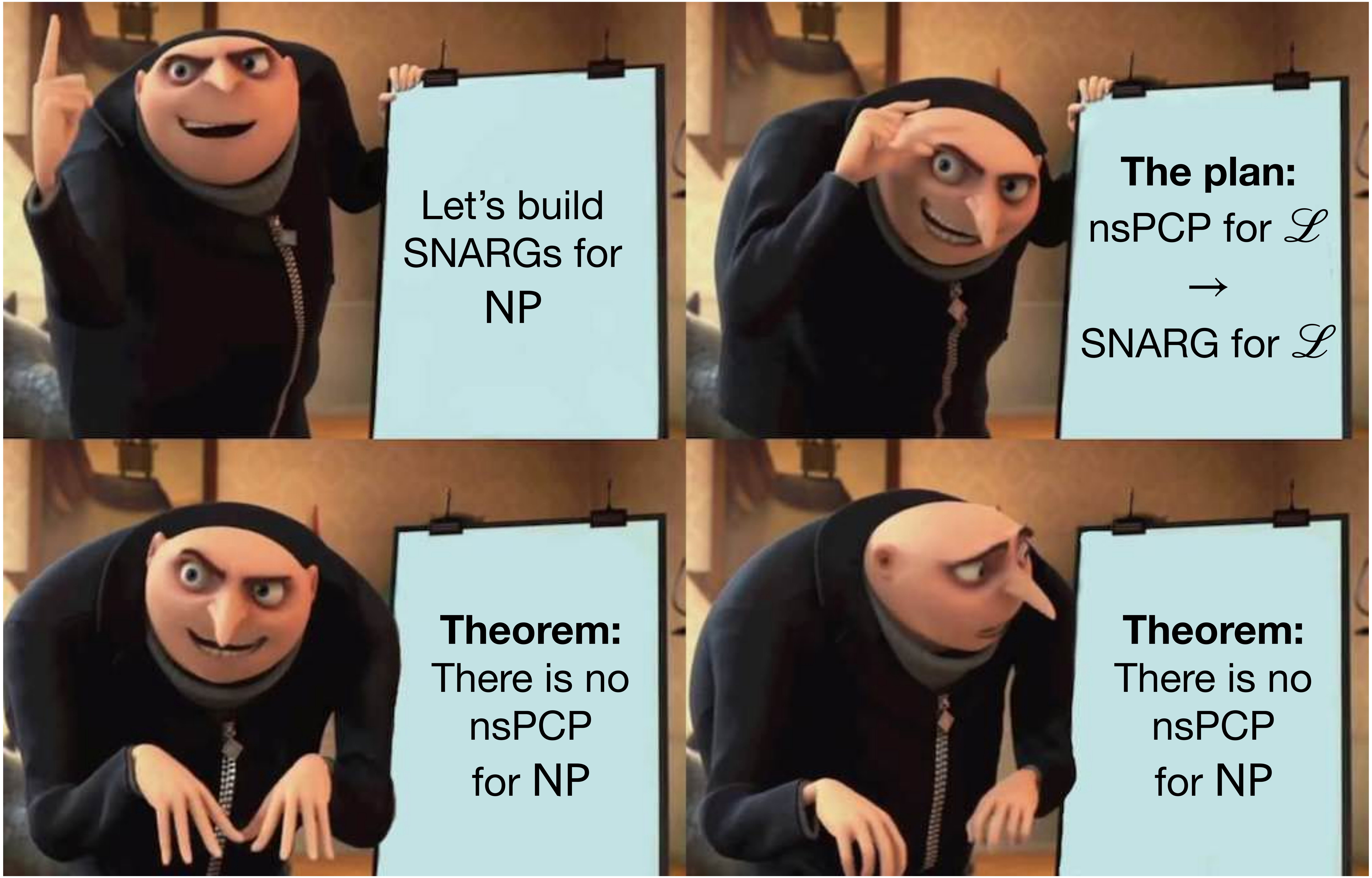
Let's build SNARGs for NP
The plan:
nsPCP for $\mathscr{L}$
$\rightarrow$
SNARG for $\mathscr{L}$


Let's build SNARGs for NP
The plan: nsPCP for $\mathscr{L}$ → SNARG for $\mathscr{L}$
Theorem: There is no nsPCP for NP



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The plan: nsPCP for $\mathscr{L}$ $\rightarrow$ SNARG for $\mathscr{L}$
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The Non-Signaling Barrier

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- **Theorem.** Assuming SAT requires $2^{O(n)}$ time, there is **no efficient NS PCP for NP**.

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#Variables in LP correspond (roughly) to all possible query sets Q to the PCP

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Our observation:

What if ℓ is $2^{O(n)}$? Then $\ell^k \geq 2^{O(n)}$.

There is no contradiction!

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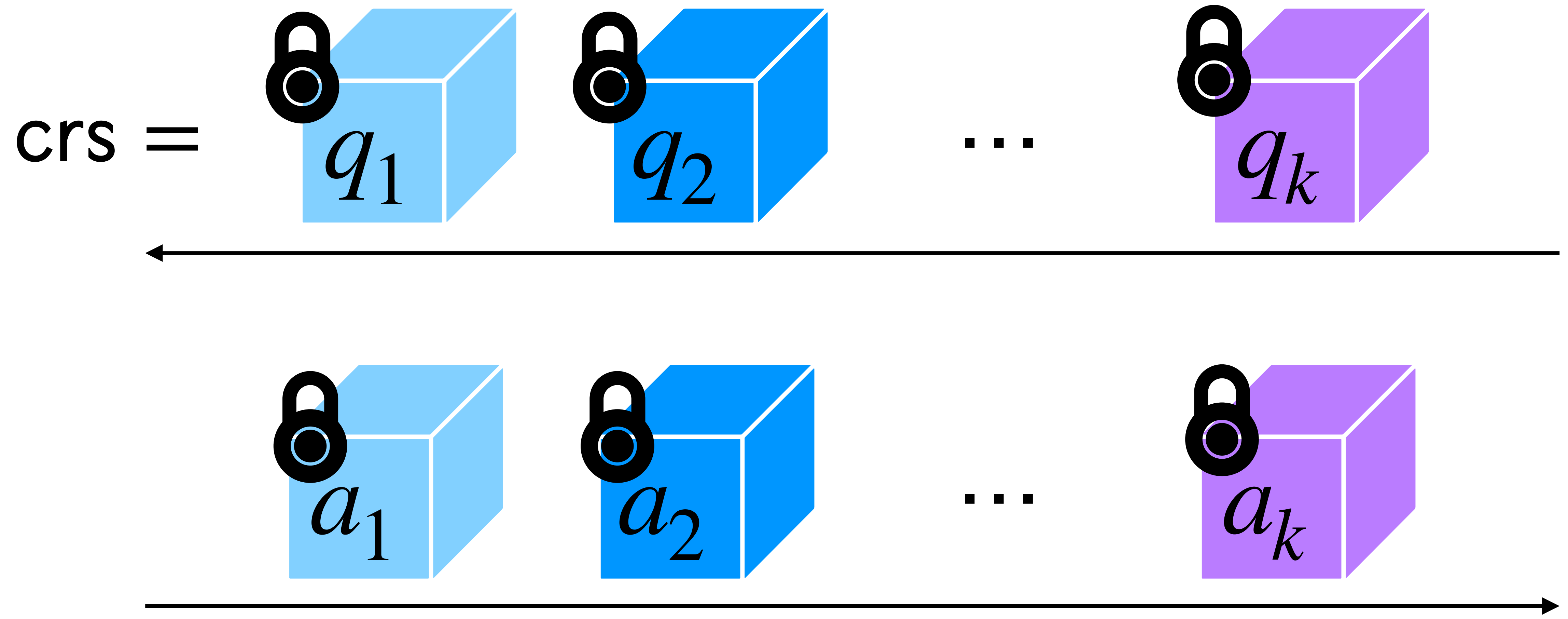

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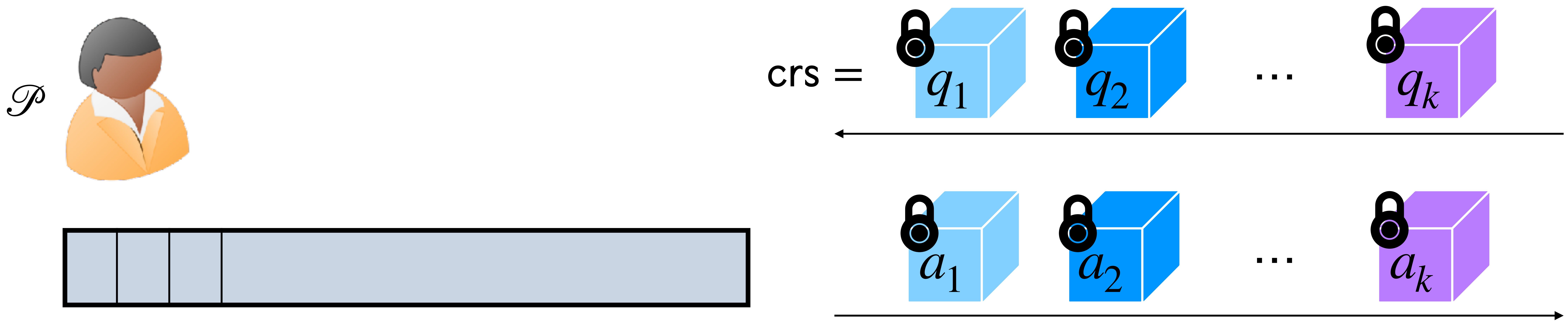

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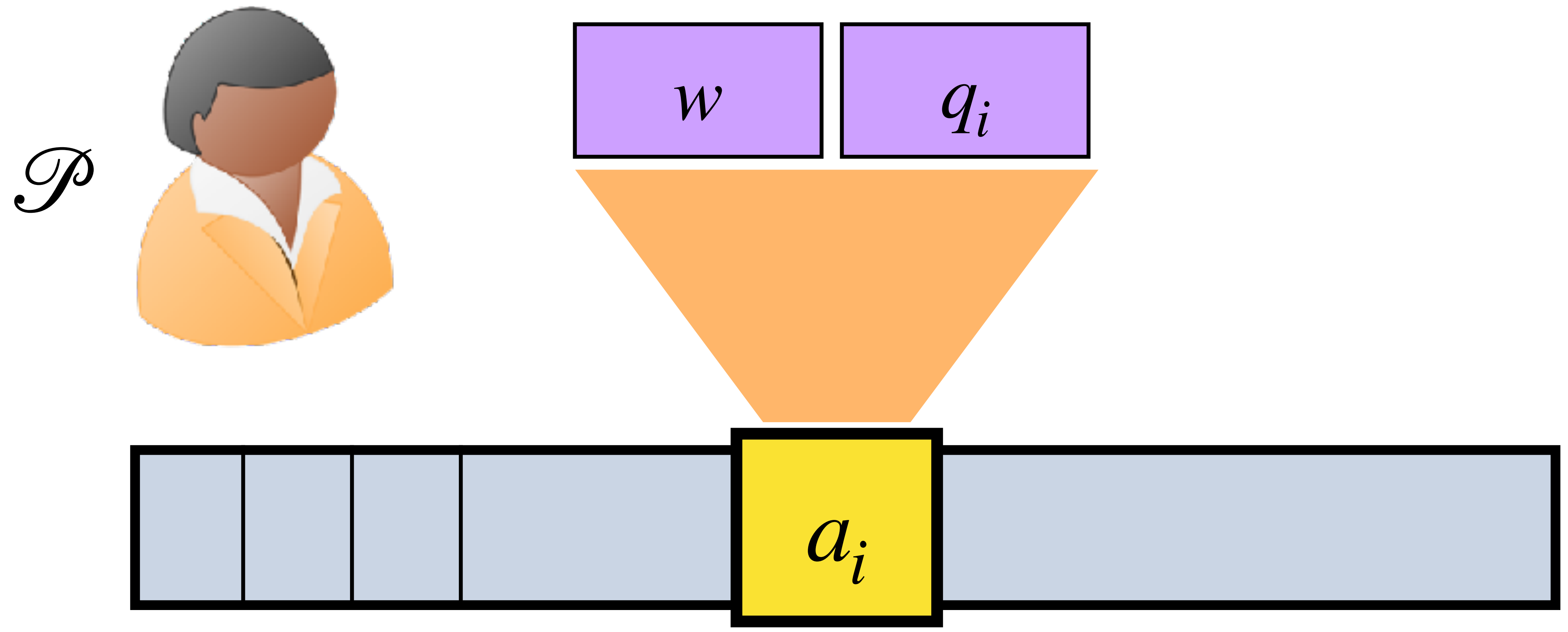

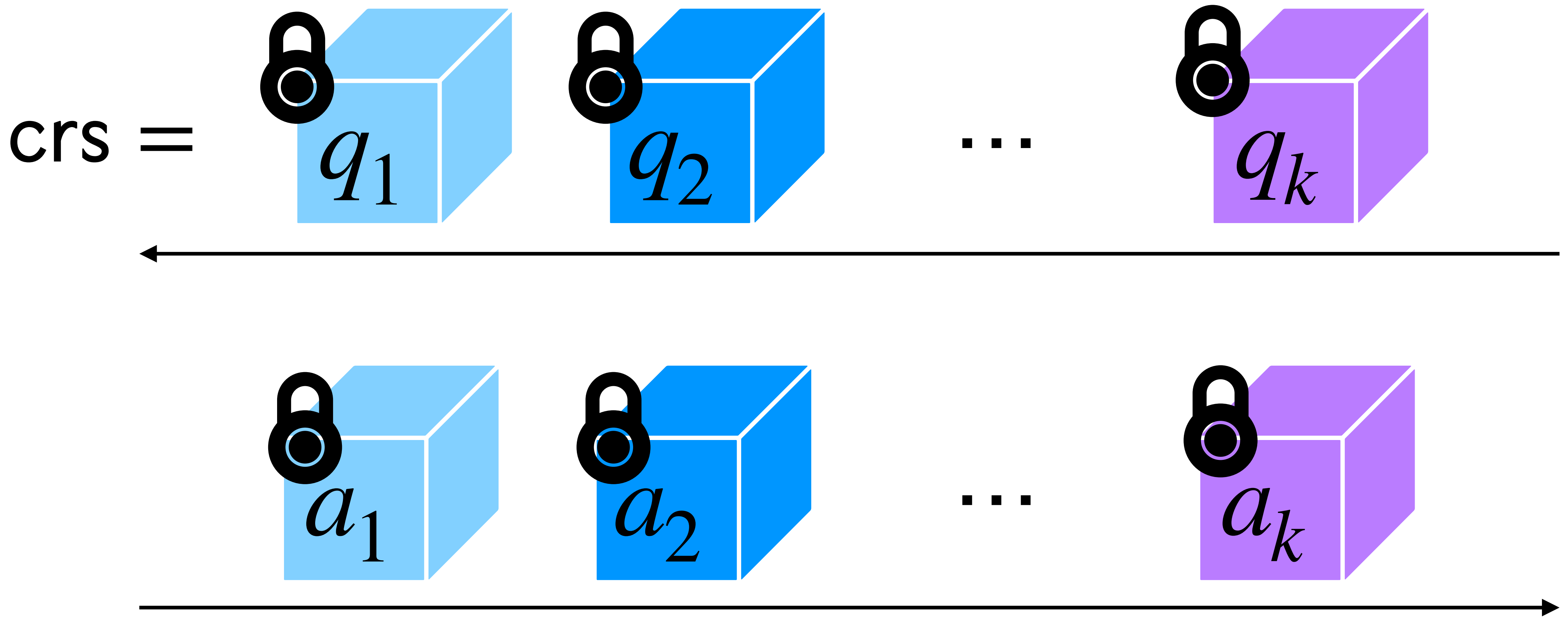

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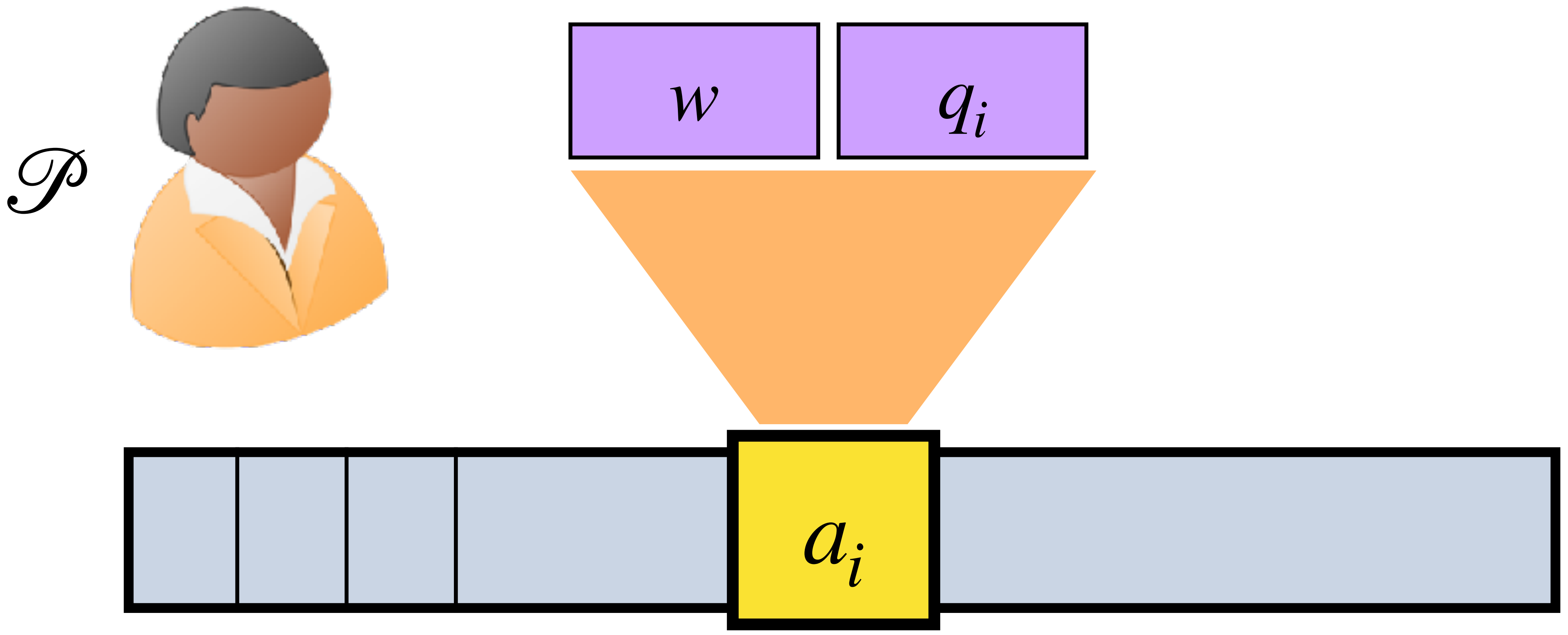

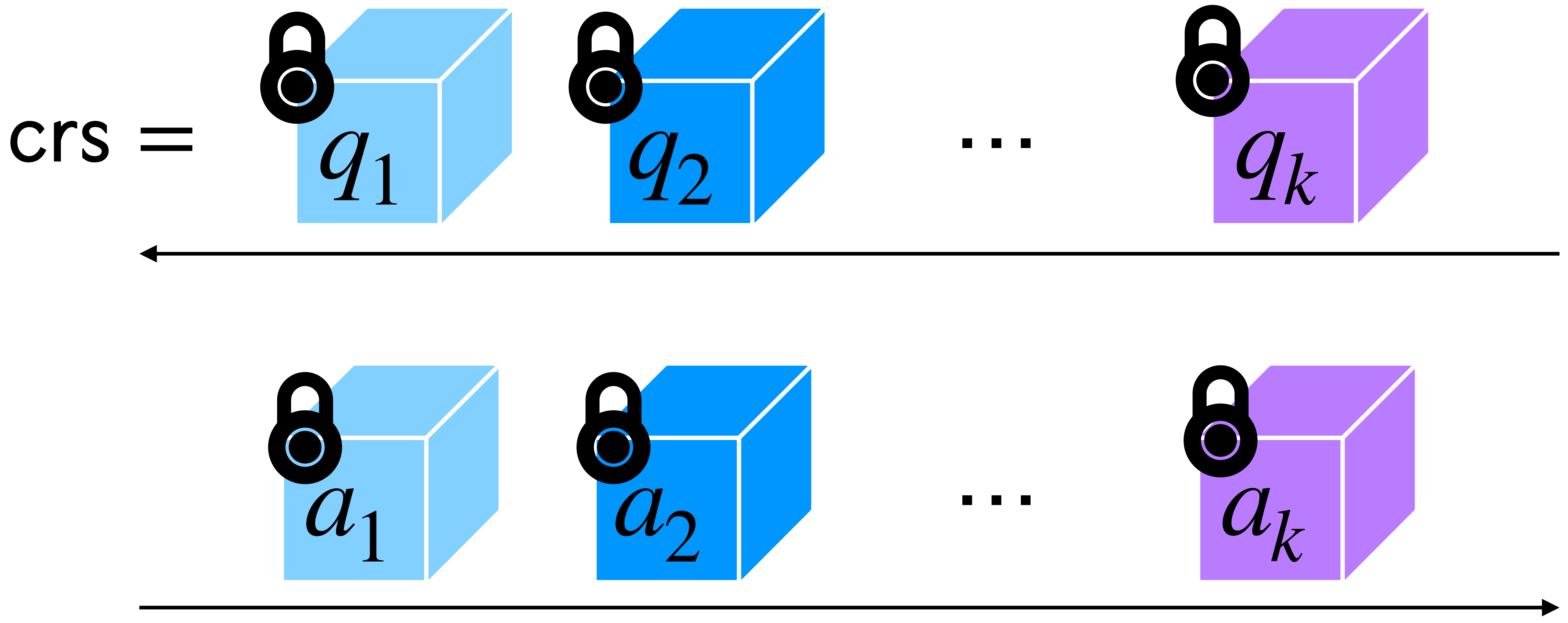

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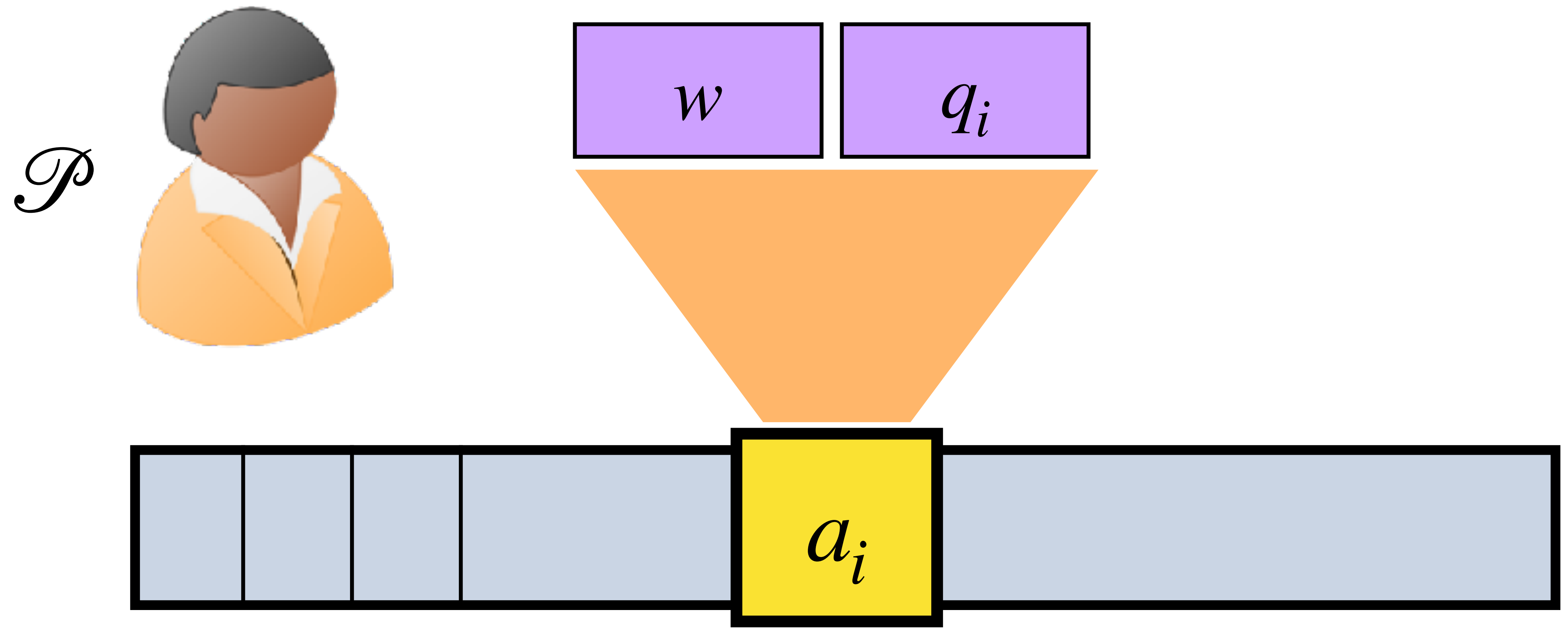

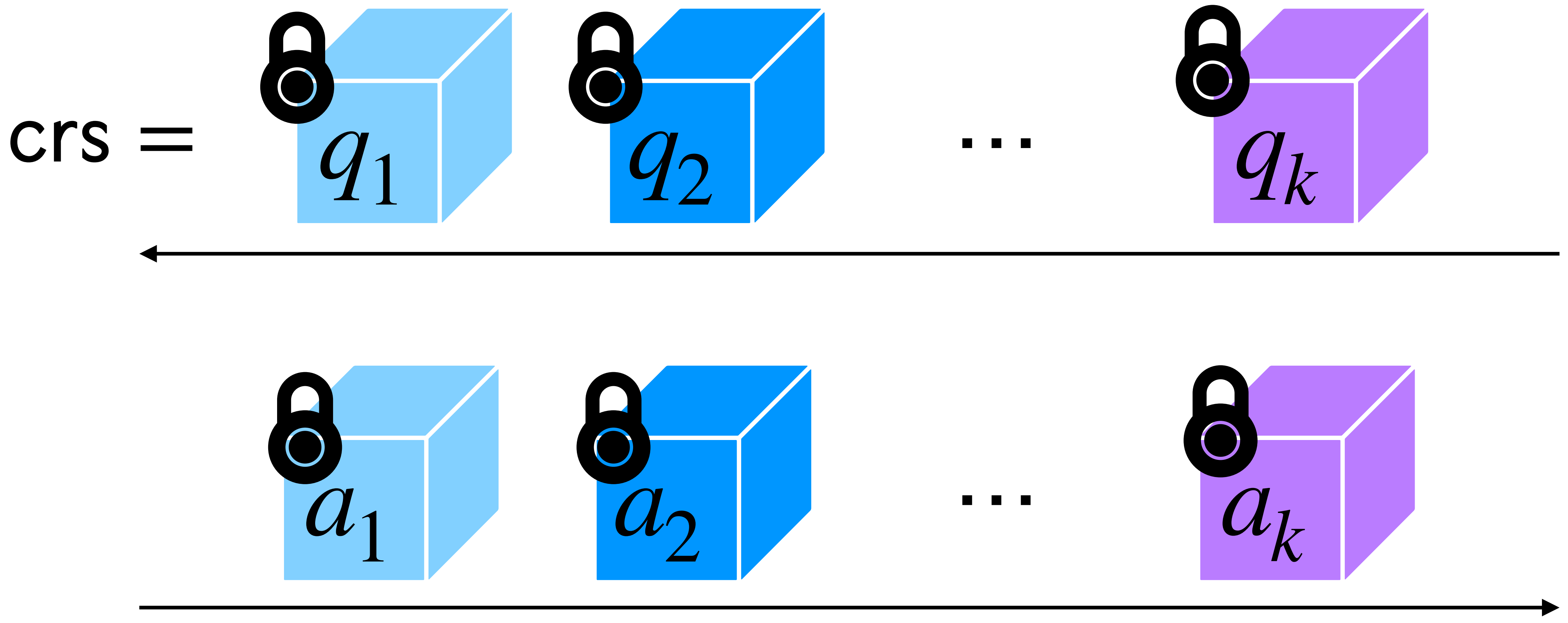

Each $|q_i| = O(n)$, $|a_i| = O(1)$

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Intuition: We show that the compiler from [JKLM25] is sound even if the underlying dvSNARG has "worst-case soundness"

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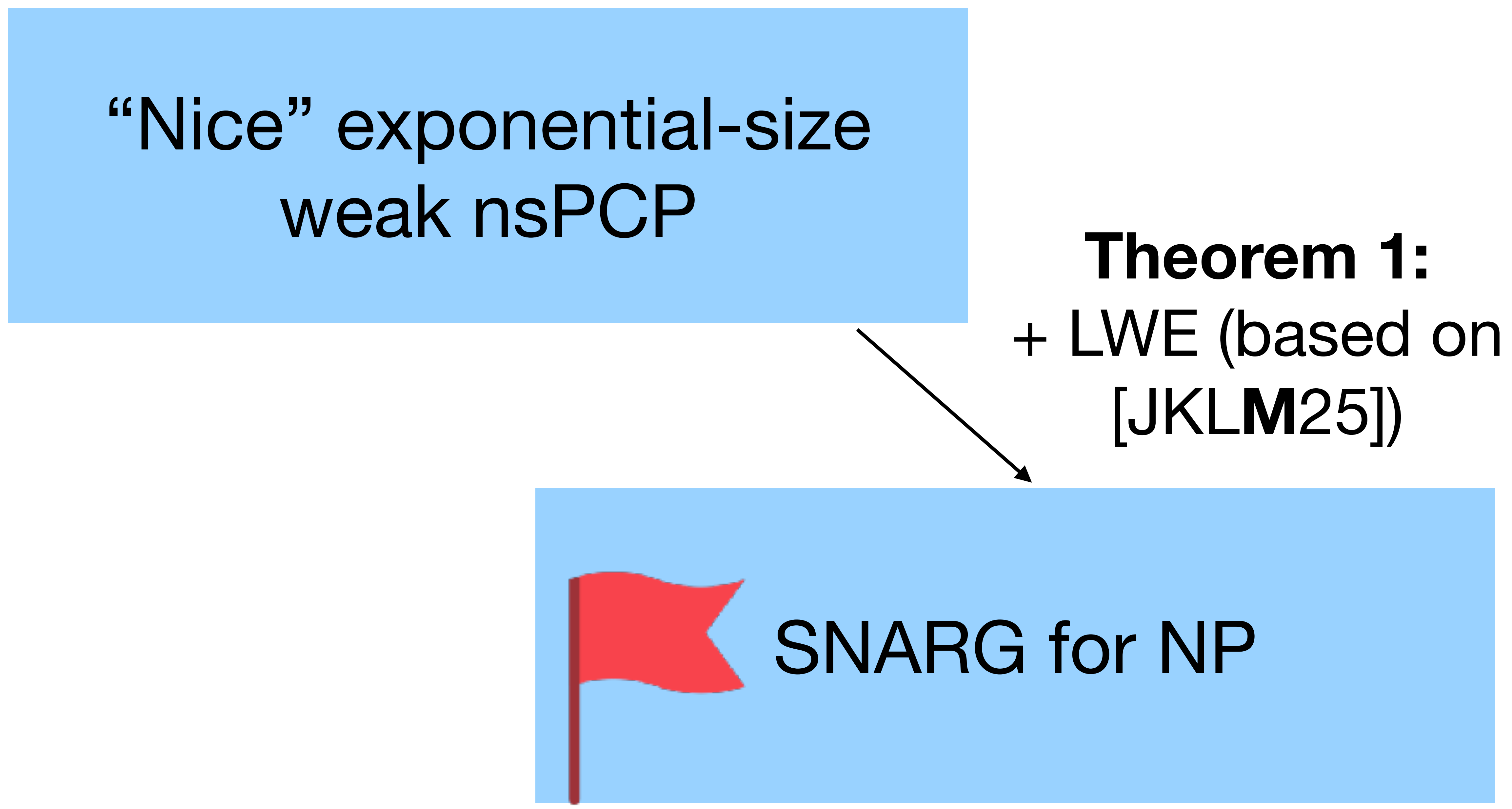

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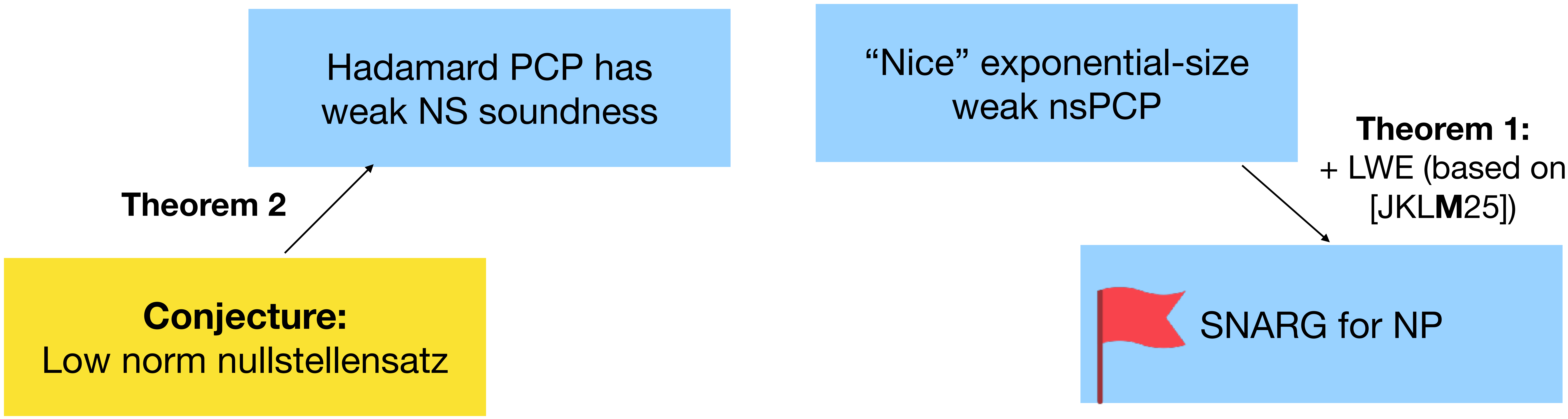

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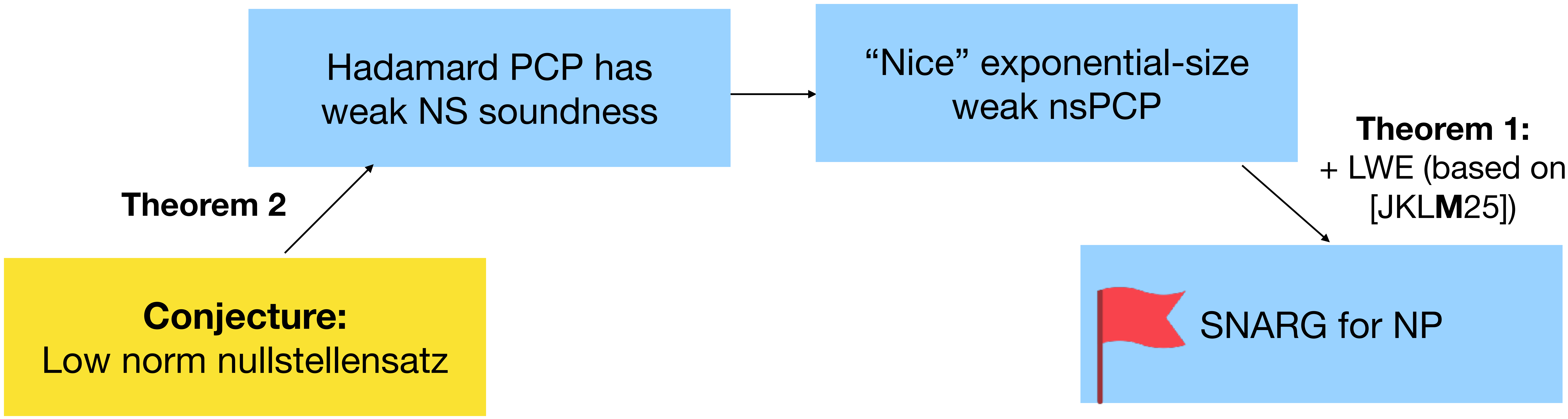

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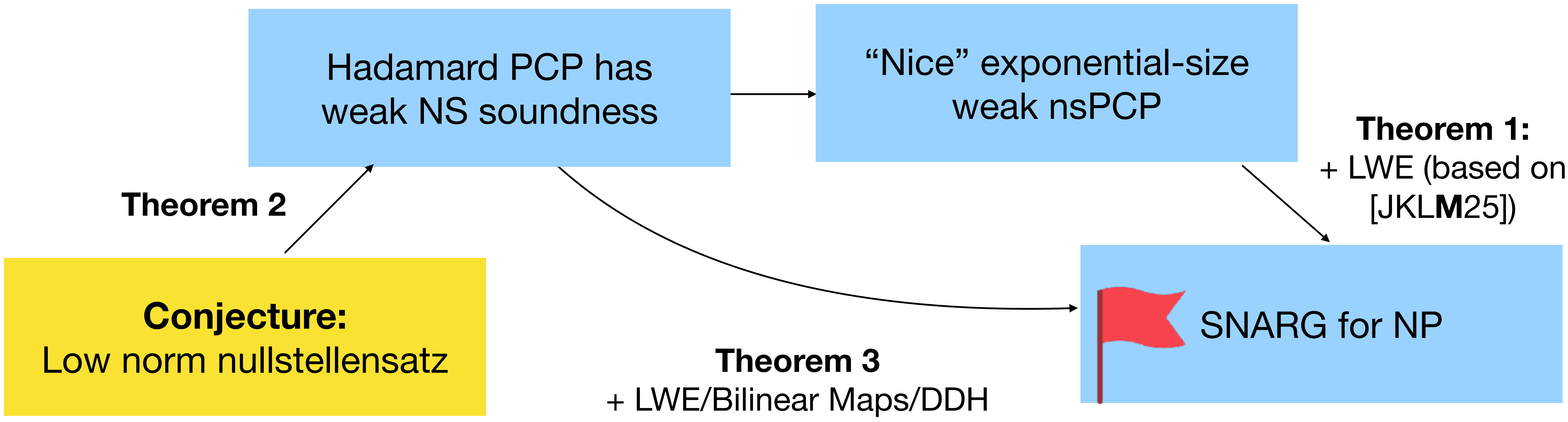

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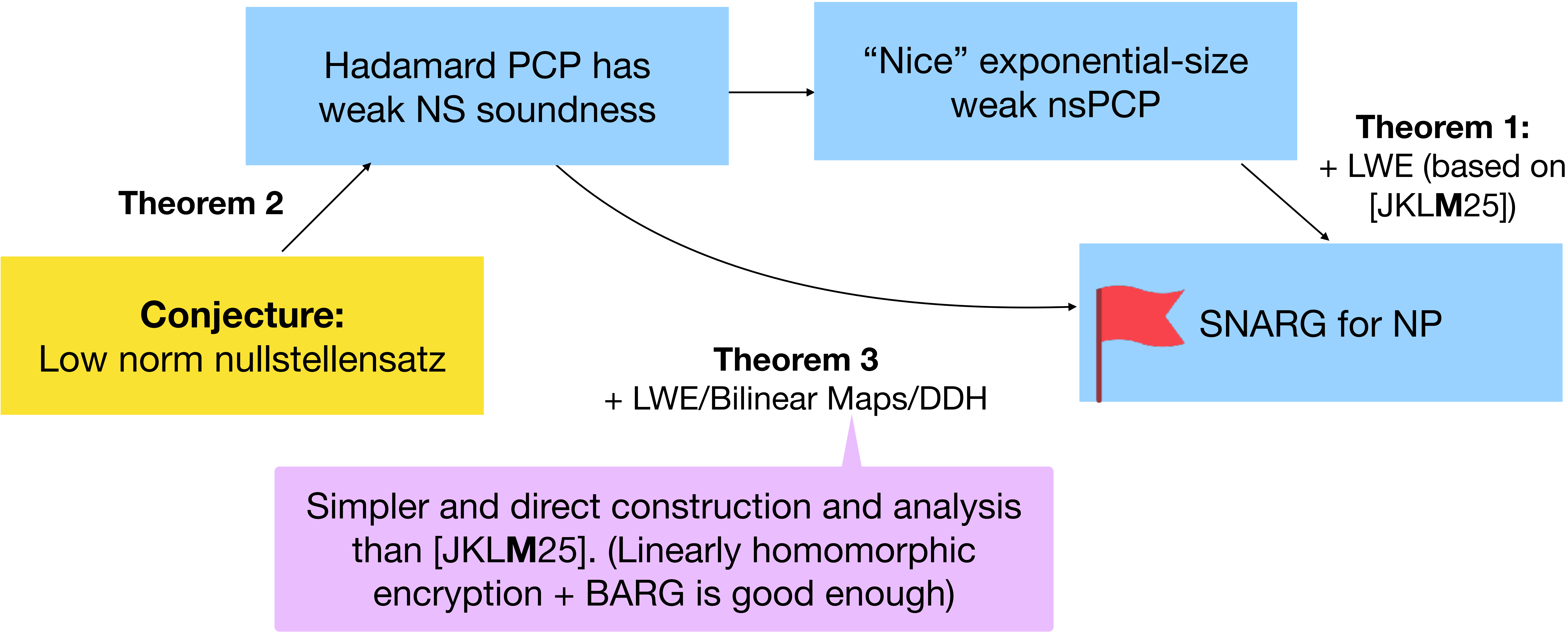

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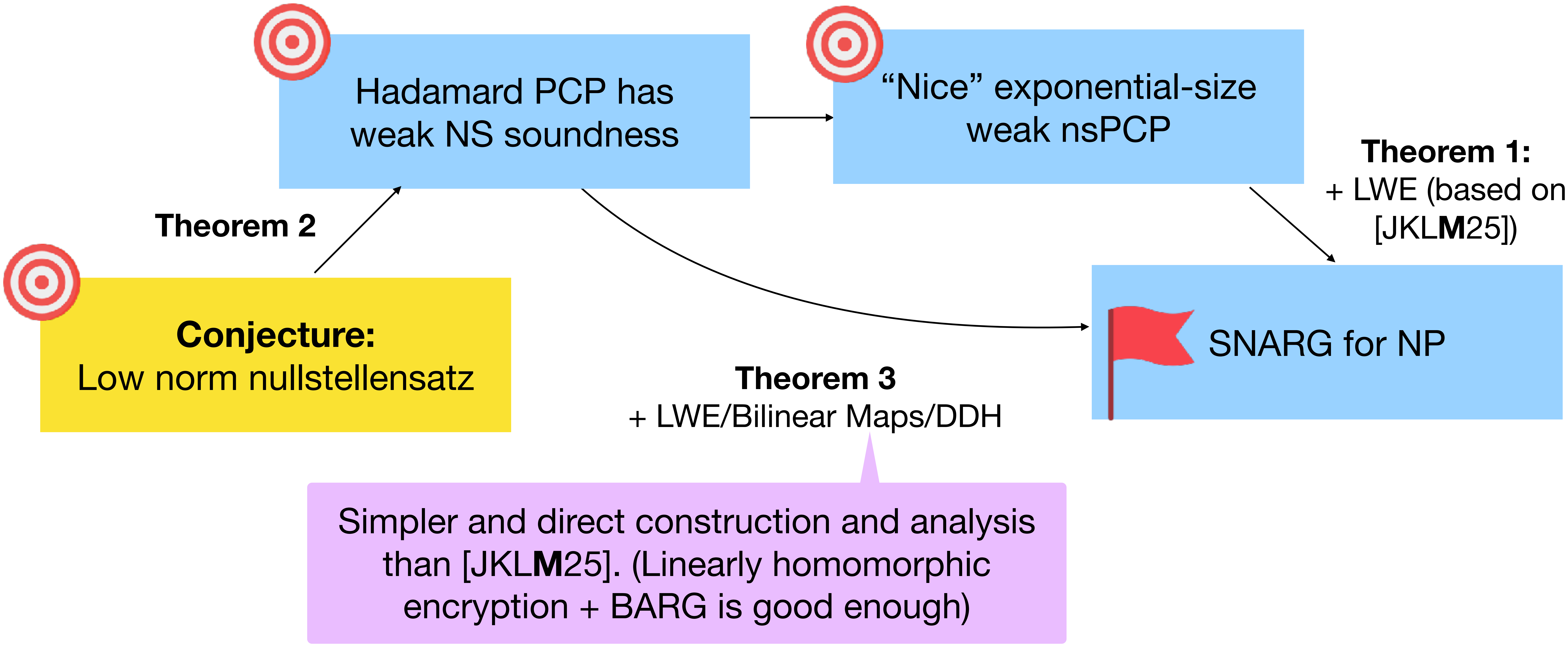

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$$w_1 - w_2 + w_{34} = 0$$

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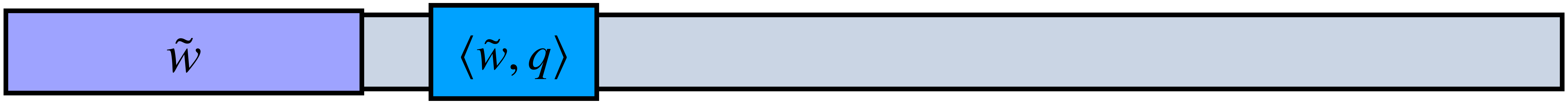

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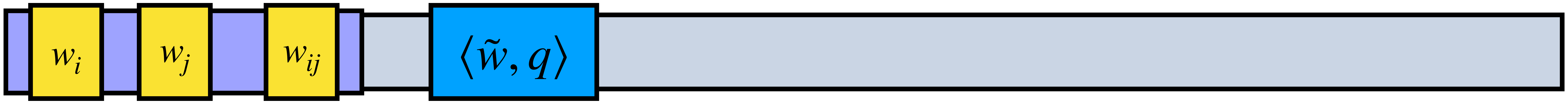

(Indicator vectors q)

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Hadamard PCP, simplified

[Babai-Fortnow-Lund '91]

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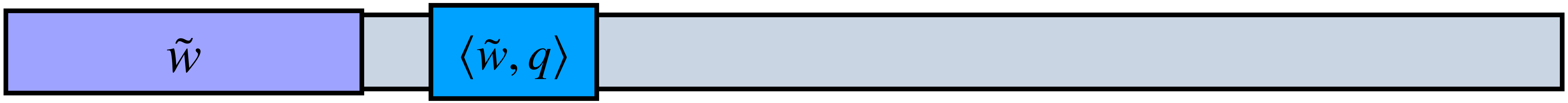

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E.g. Contains
$w_j - w_j + w_k = 0$

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…

p_{2^n} Had($\tilde{w}_{2^n}$)

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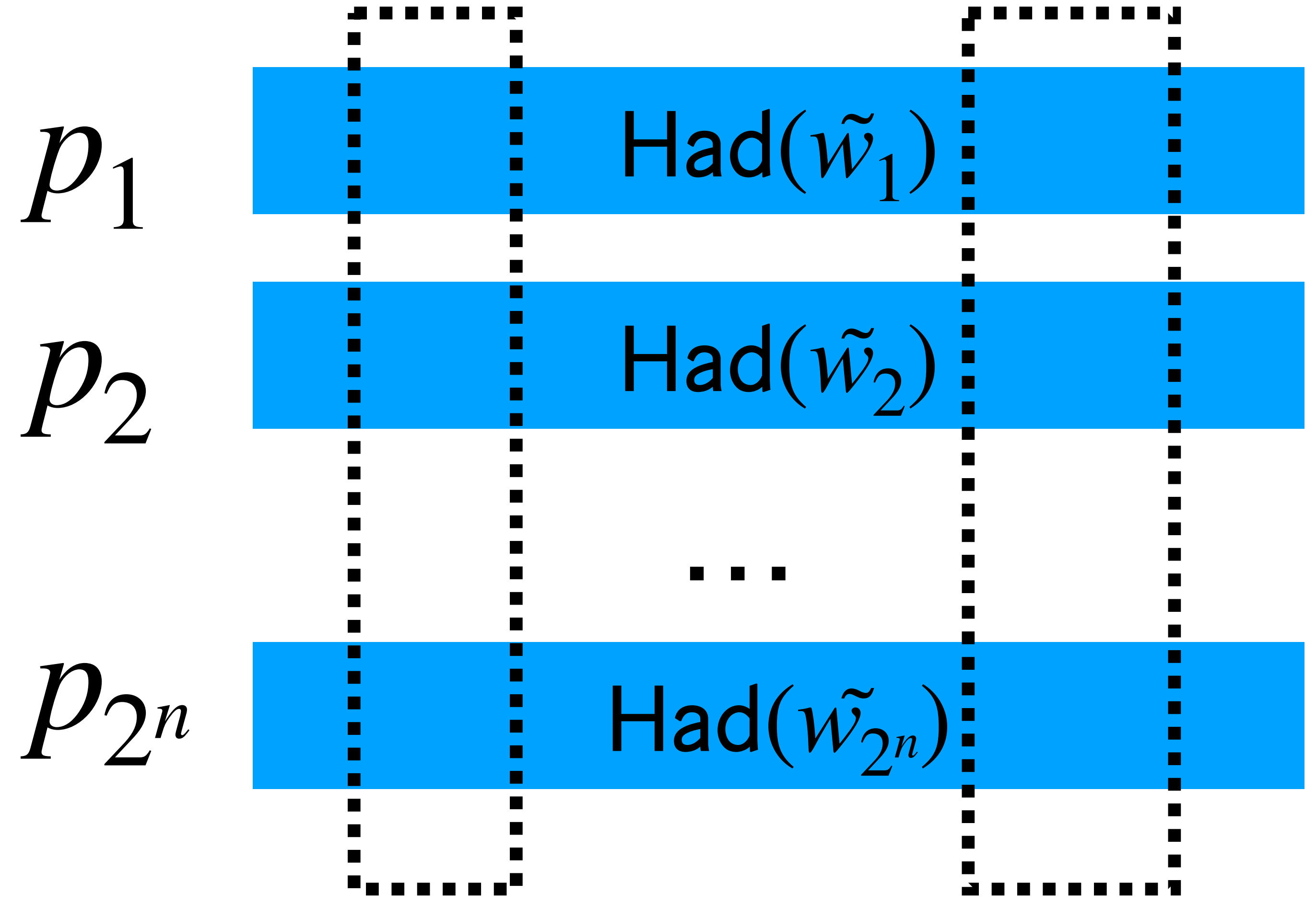

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Real Proof Sketch

Extreme Bird's Eye View

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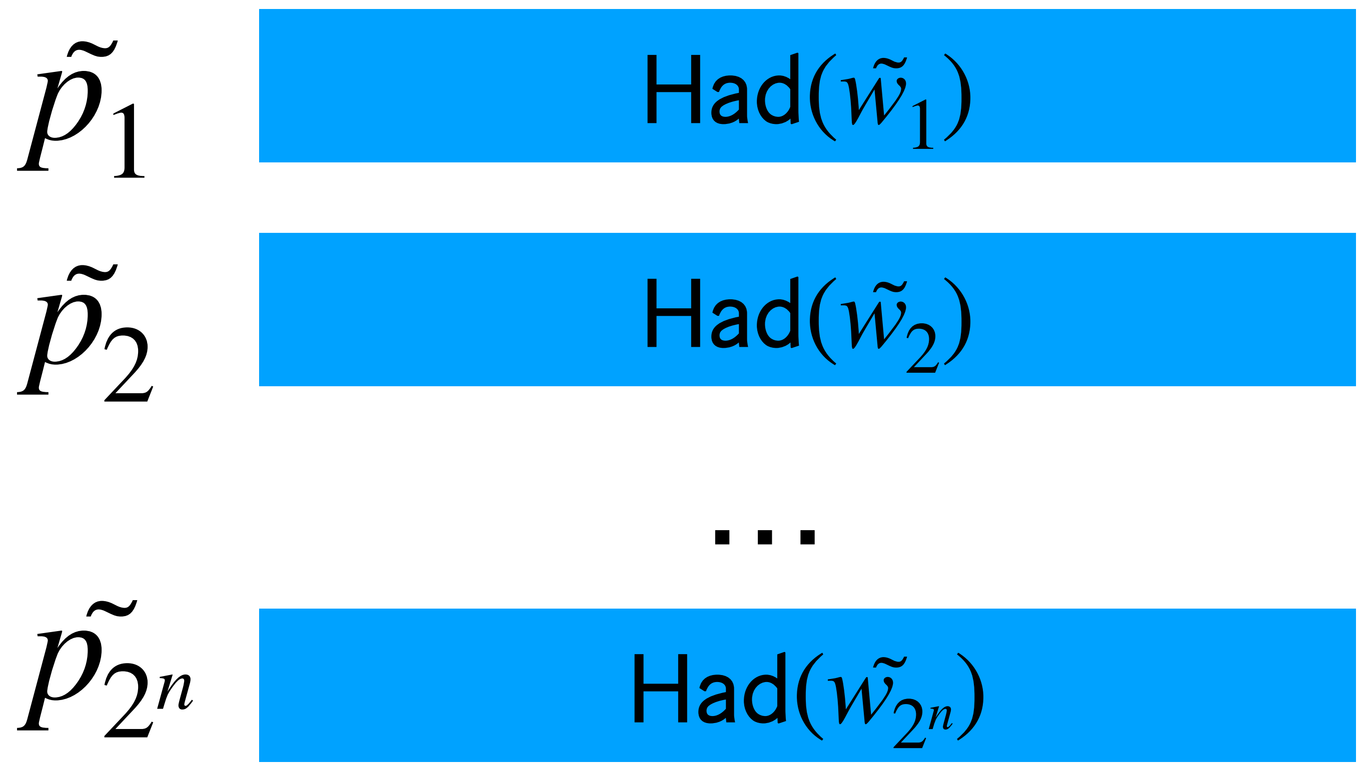

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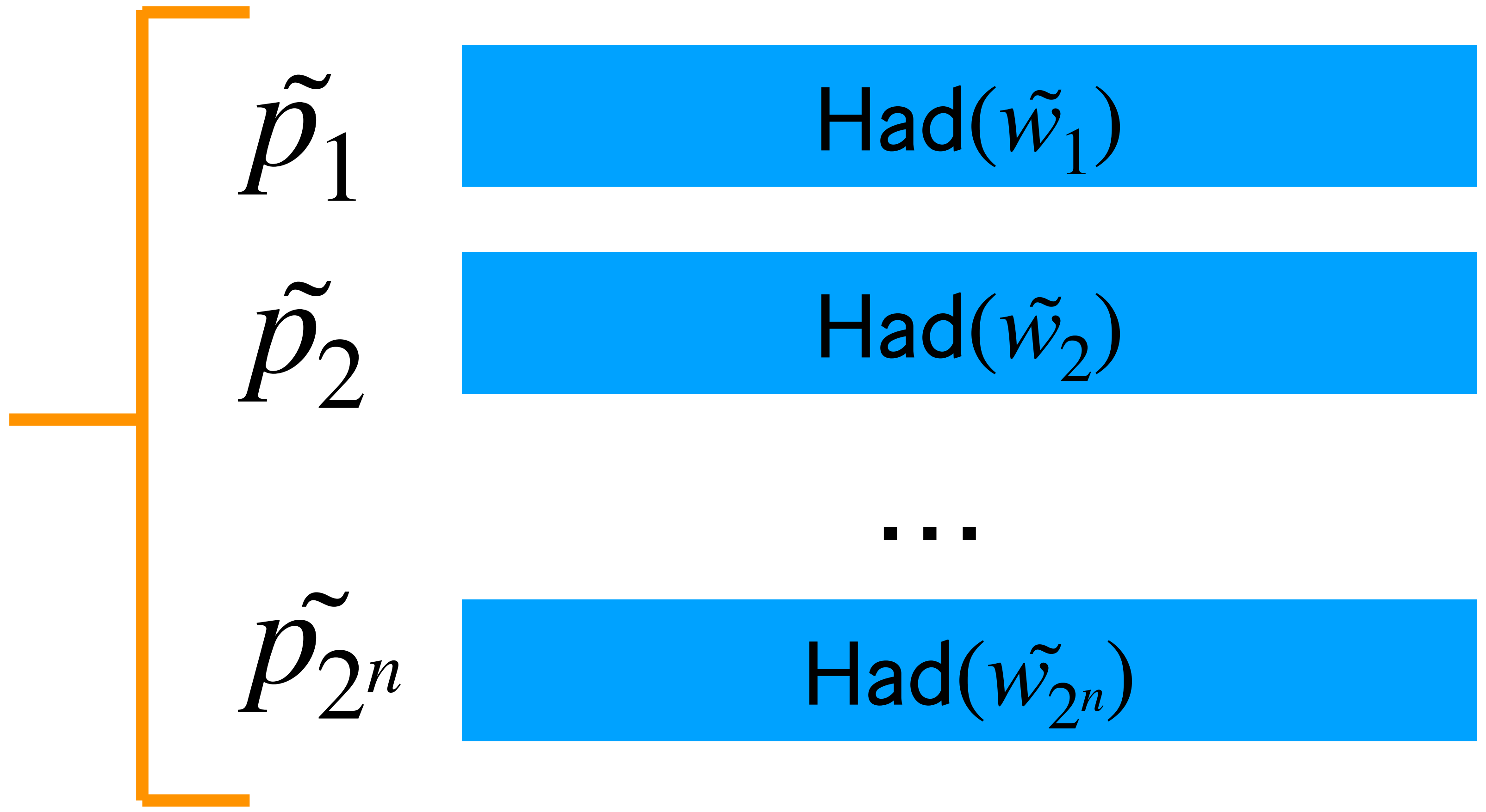

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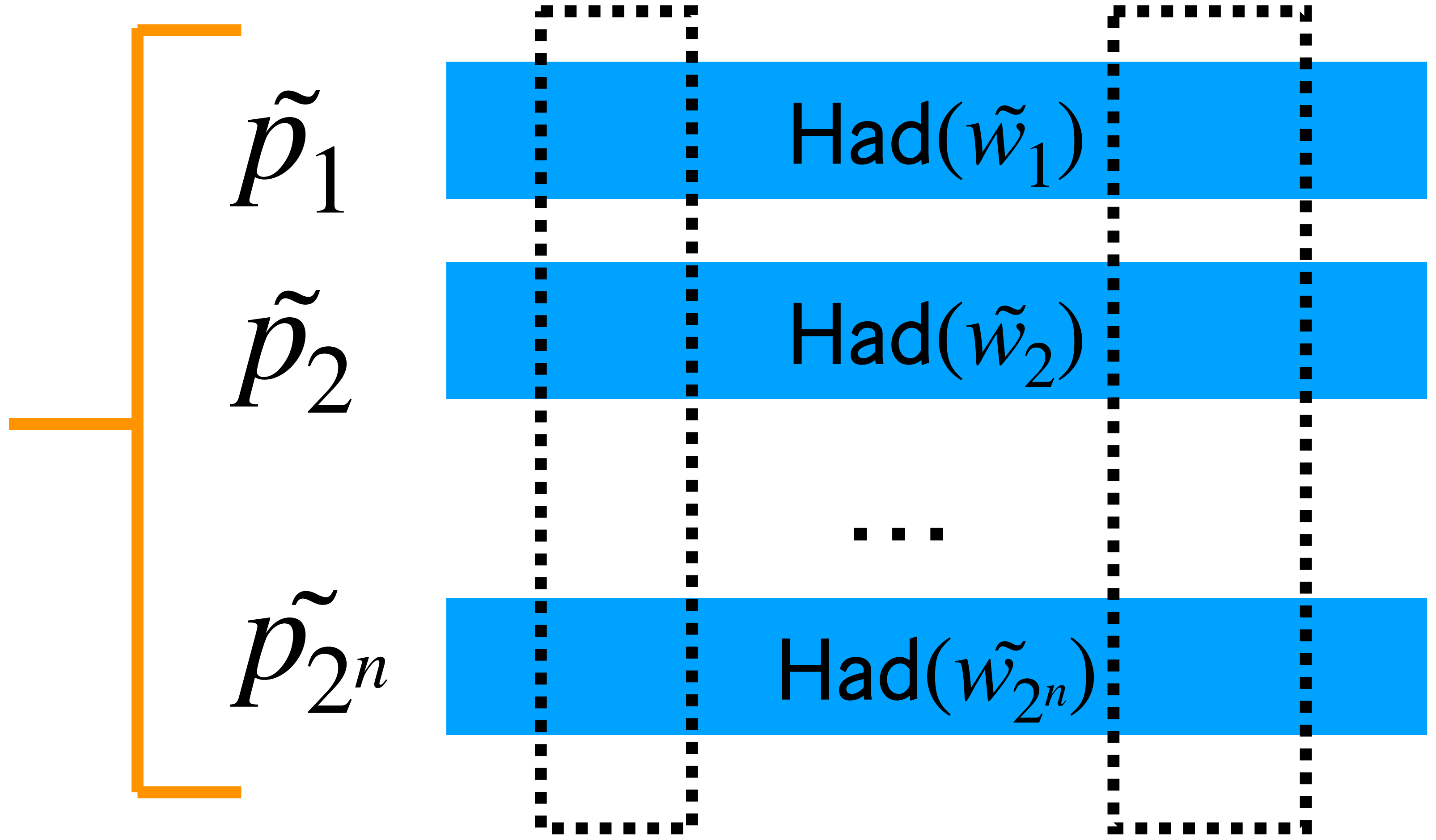

D_Q corresponds to the marginals.

Real Pro

Extreme B

Uses Hilbert's Nullstellensatz
and Sherali-Adams pseudoexpectations.
Uses ideas from [CMS '18]

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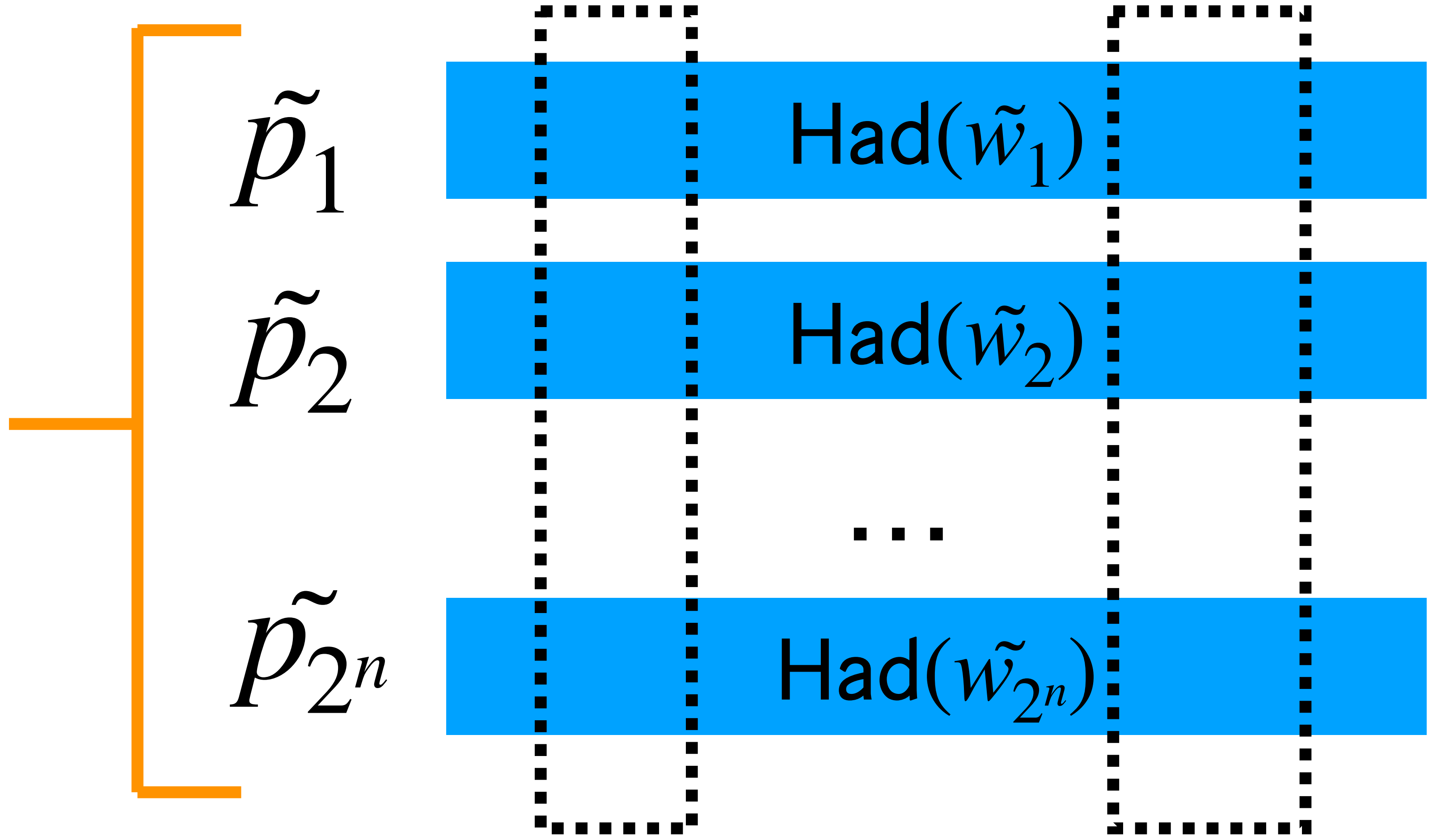

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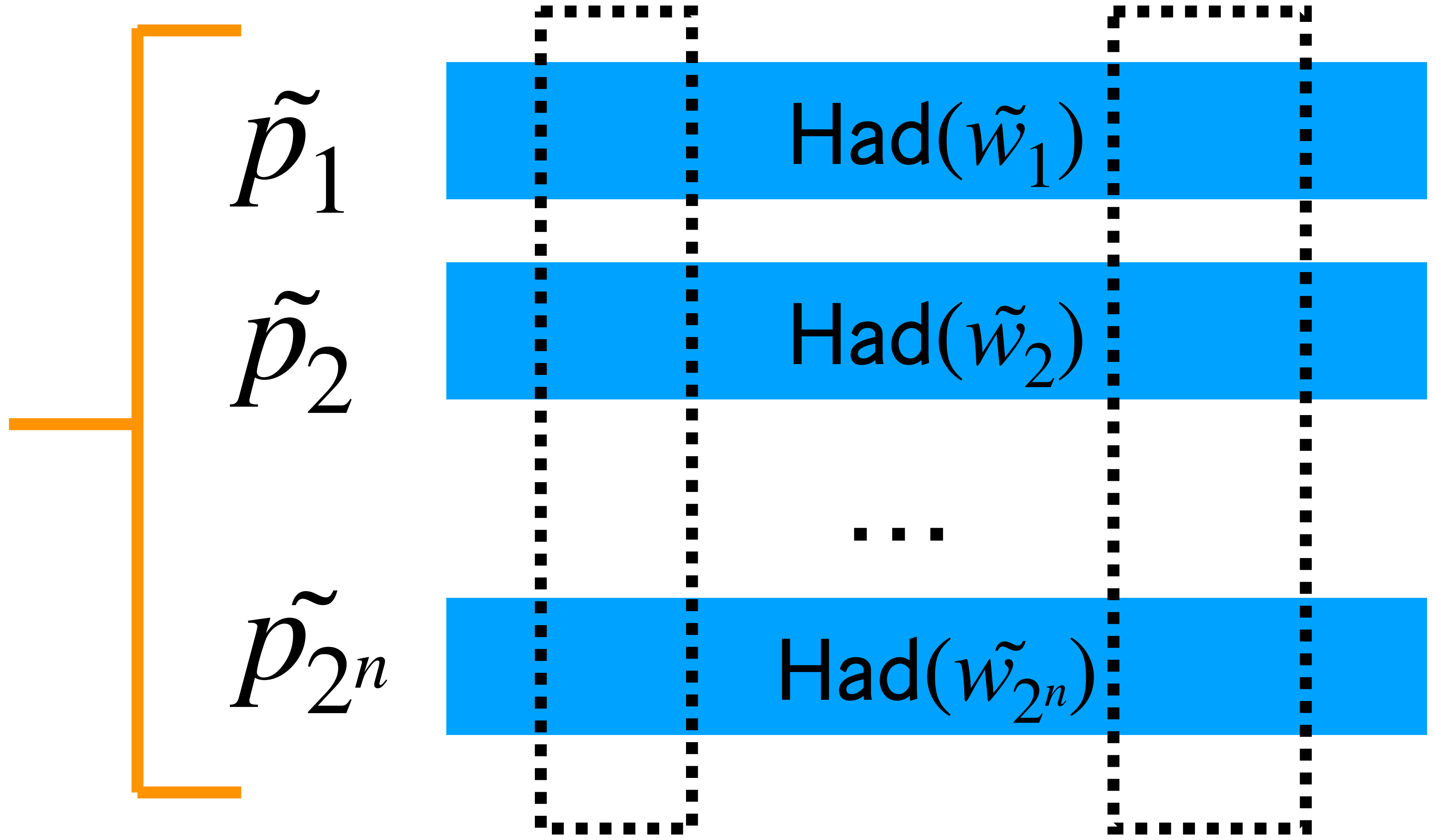

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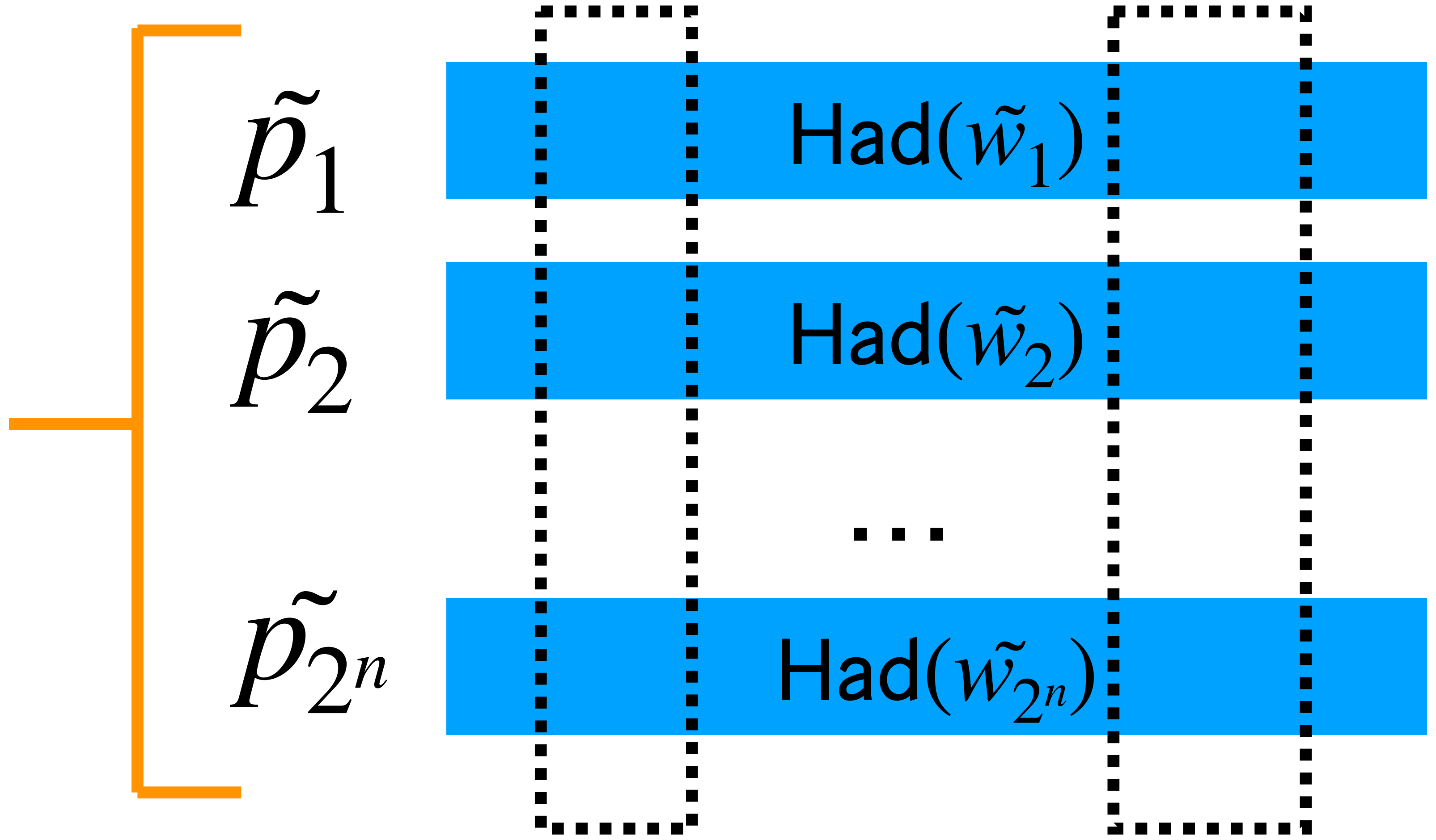

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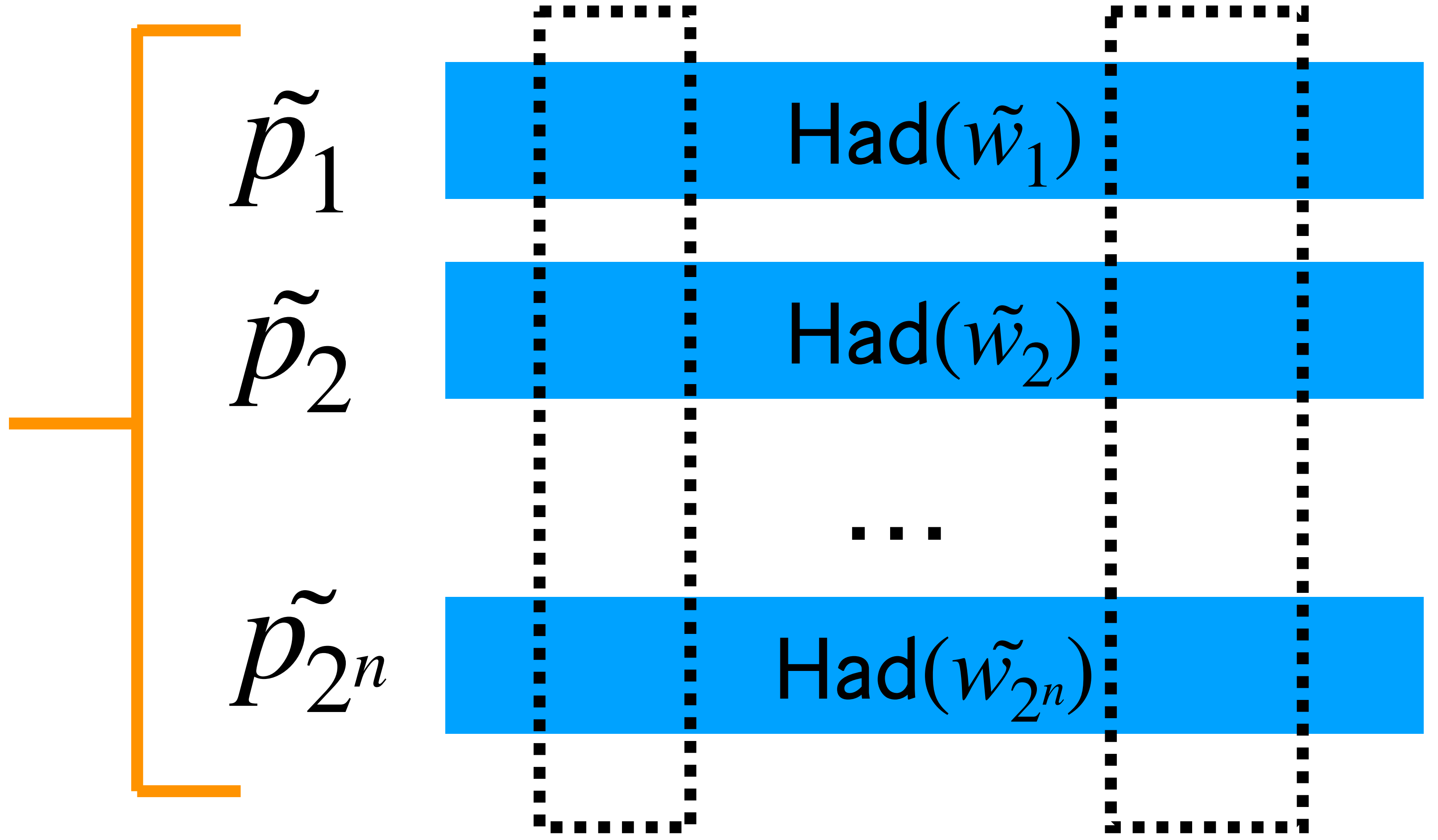

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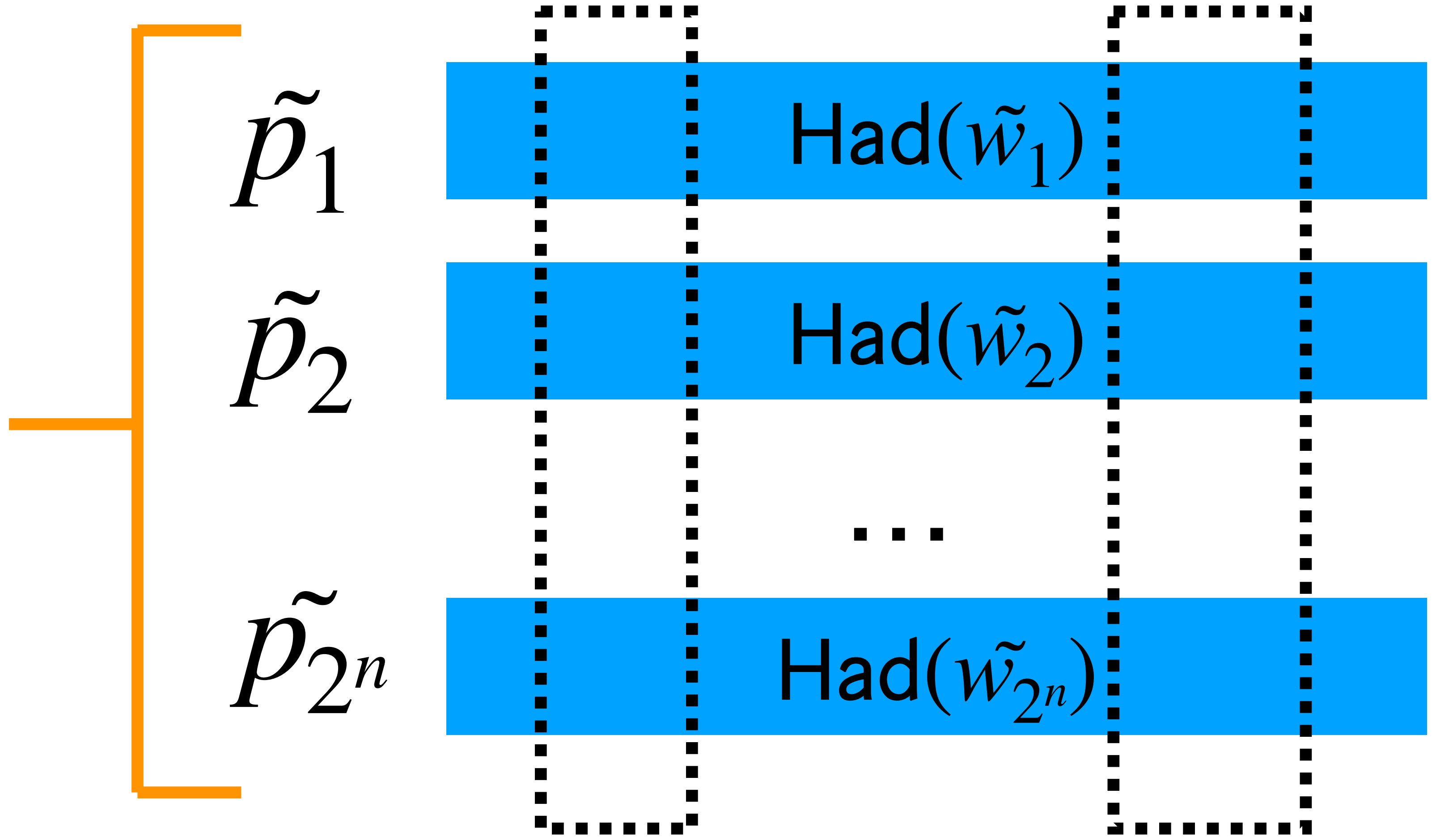

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Local views look "real": These probabilities will be in [0, 1]

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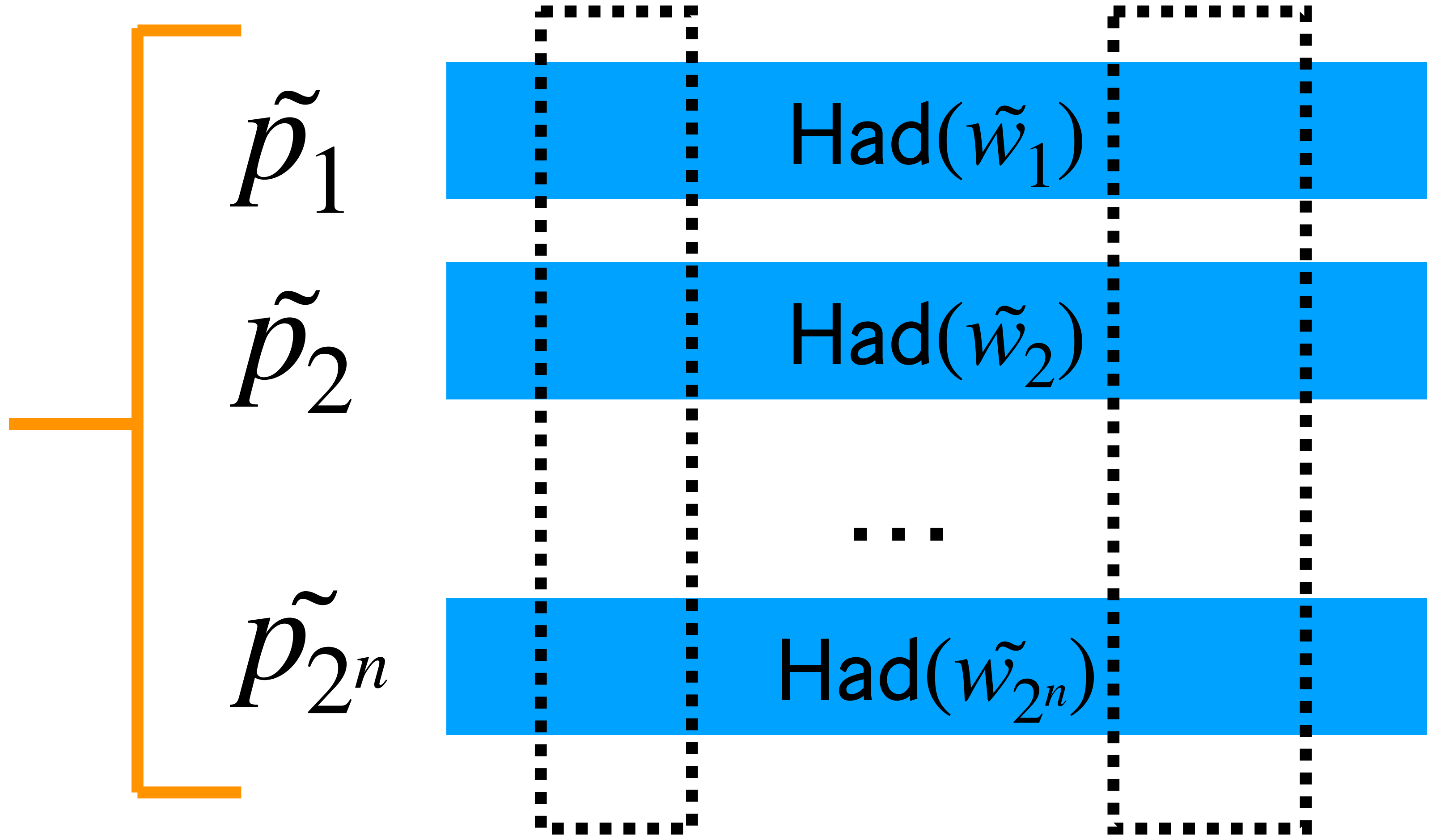

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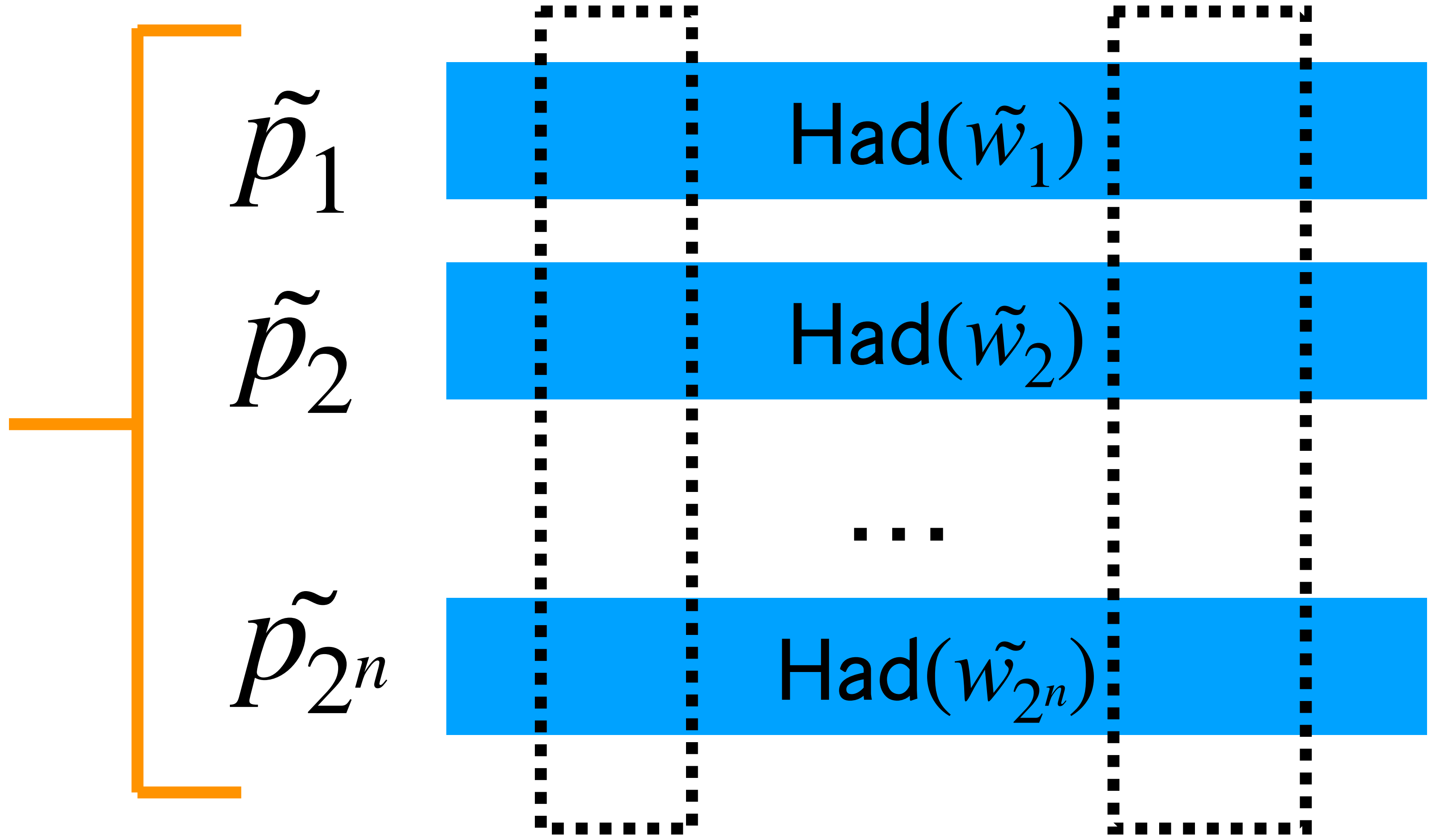

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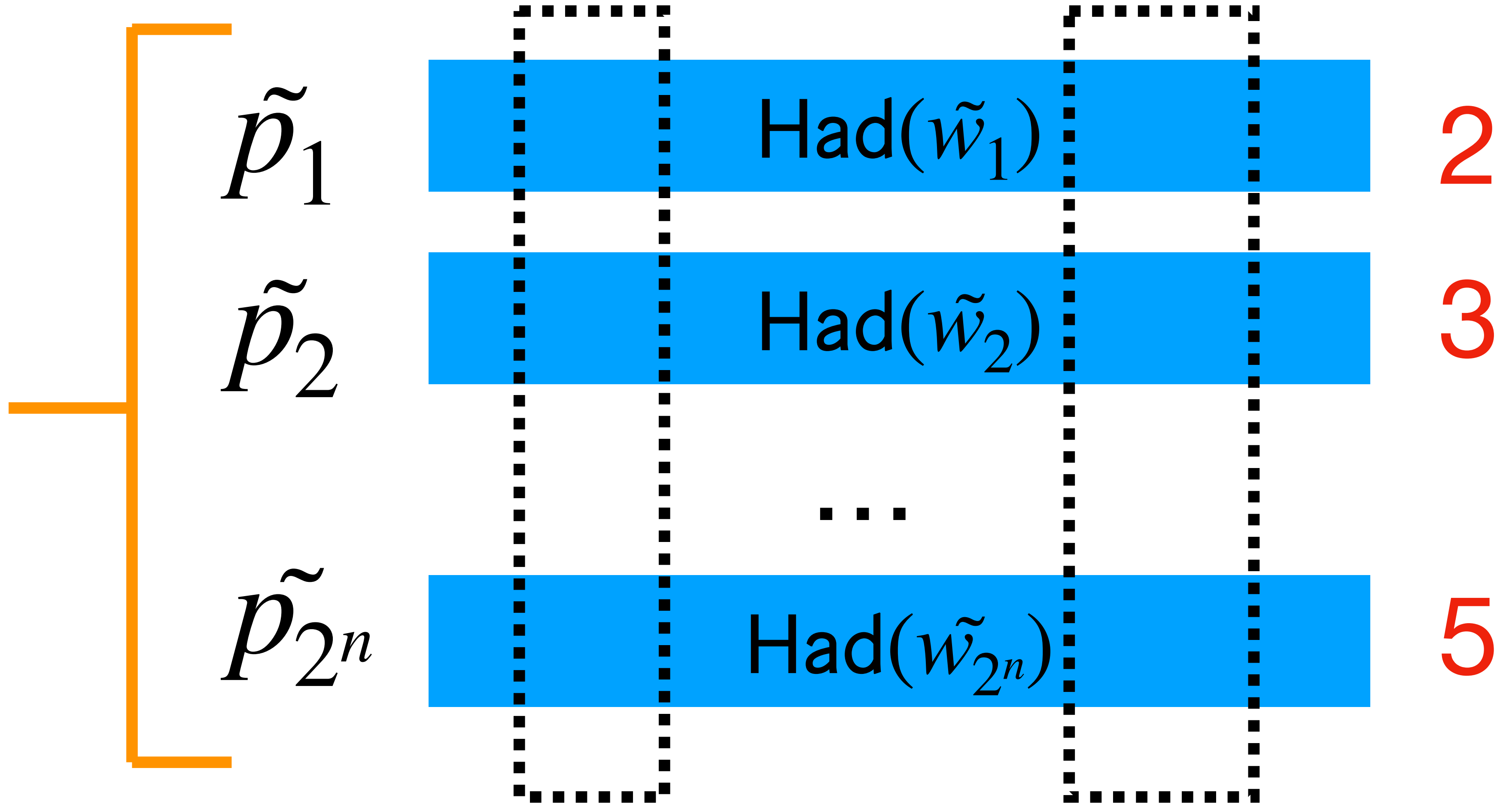

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- **Step 2:** If $x \notin \mathscr{L}$, then every encoding fails *some* **satisfiability test.**
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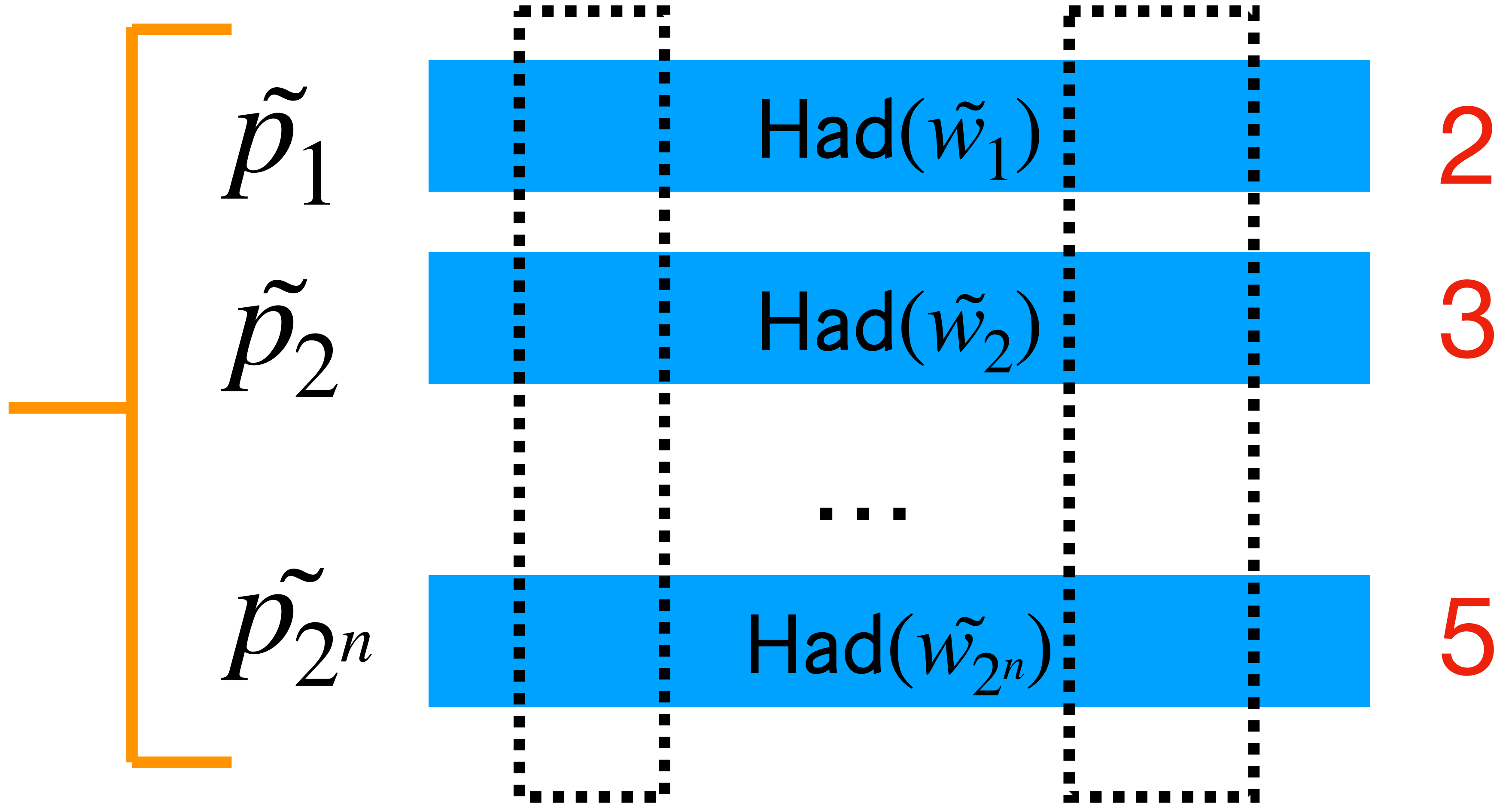

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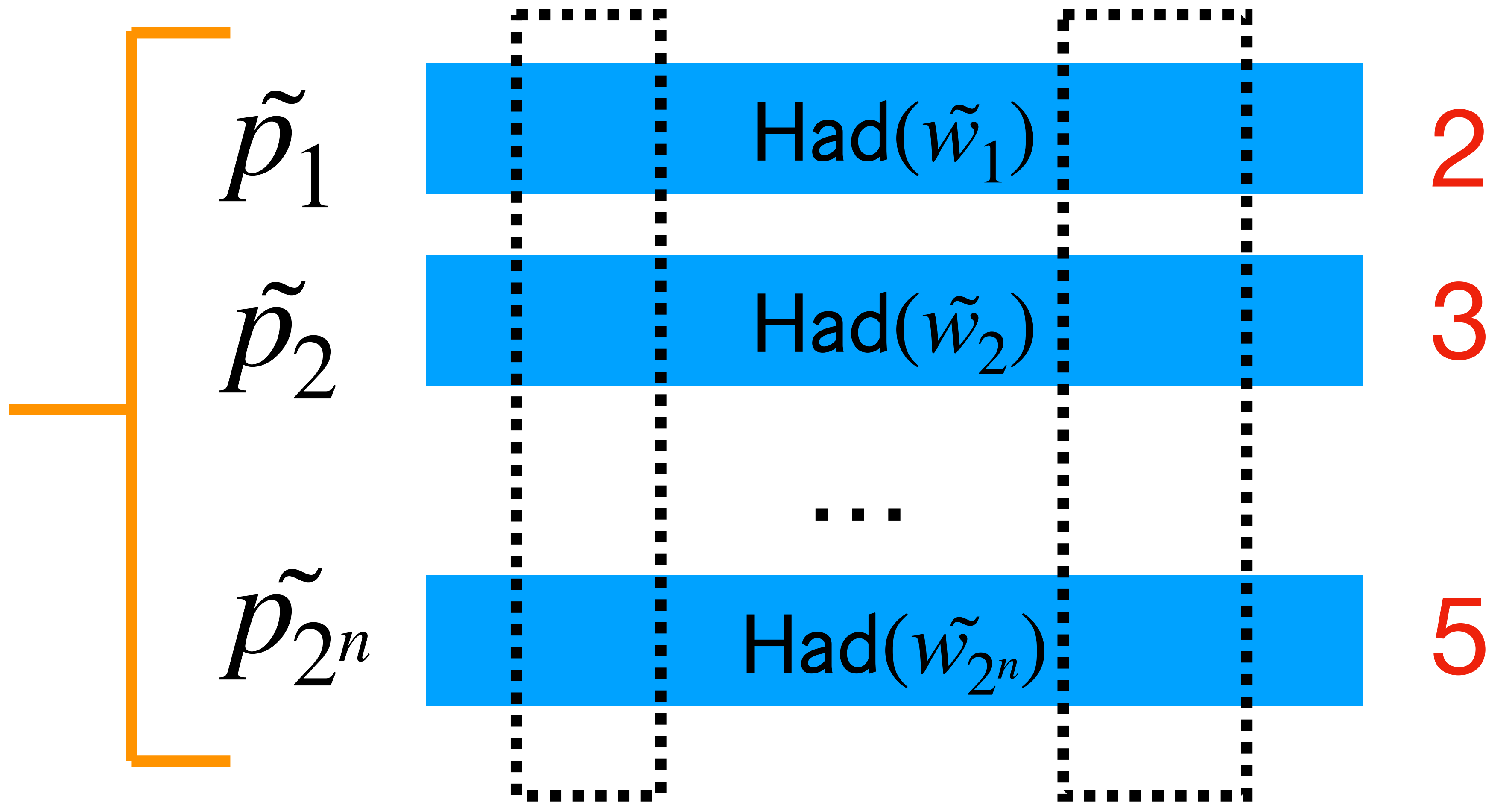

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Fix (high-level):
Use Hadamard encoding to read "**random** linear combinations of the **satisfiability tests**".

Use careful counting.

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- We relax the requirement in two ways:
  - We allow for **exponential-length** PCPs.
  - We relax to soundness requirement to **“weak soundness”**.
- **Candidate construction** under a conjecture.
- **Open question:** Does there exist a NS PCP with weak soundness?

Thank you for your attention!

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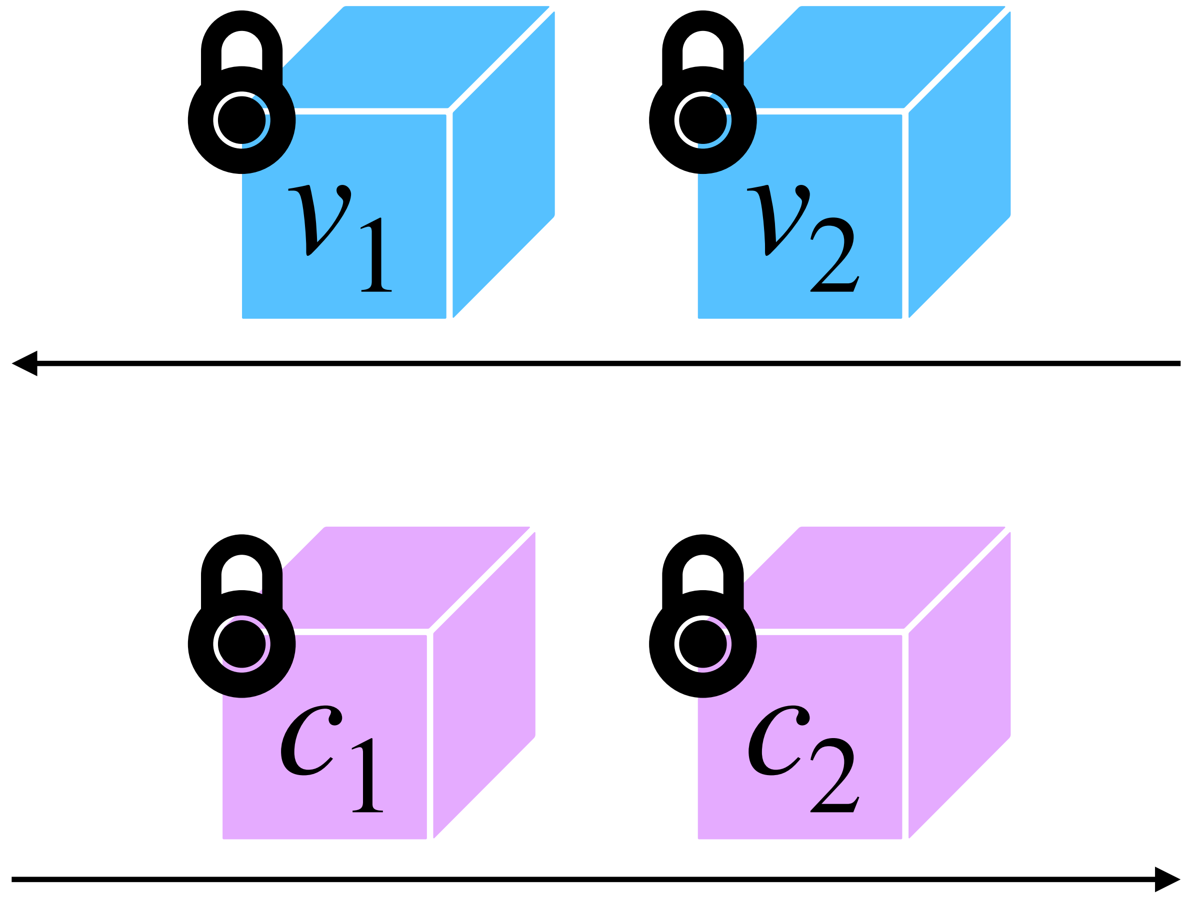
Images used are from flaticon.com, tikzpeople,

Oversimplified counterexample

[Dwork-Landberg-Naor-Nissim-Reingold '04]

- **Language:** Graph 3-Colouring
  - **PCP string:** 3-colouring of the graph
  - **Verifier:** Check a random edge (v1, v2). Catches with probability $1/|E|$.



Issue: Prover is not "committed" to any single PCP string!